

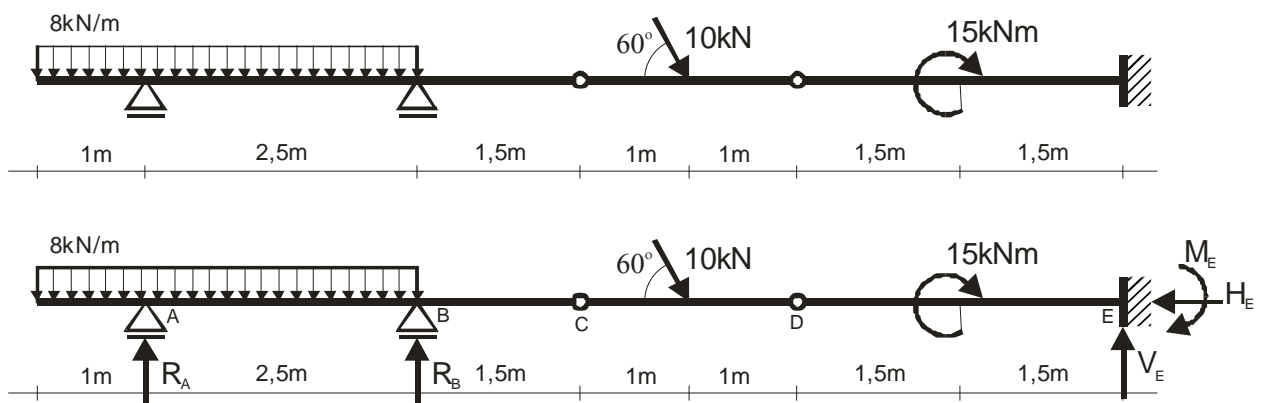
Mechanika ogólna

Wykład nr 6

Obliczanie sił wewnętrznych w belkach – przykłady

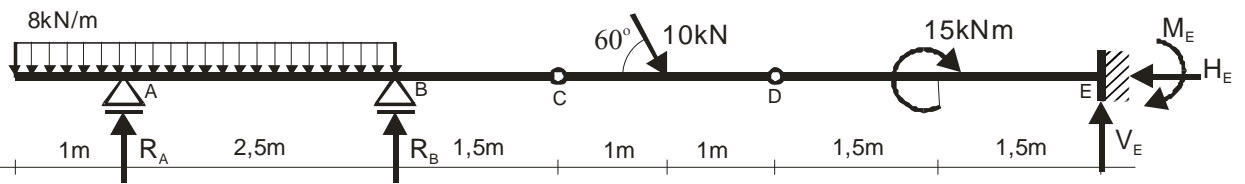
1

Przykład – belka przegubowa



2

Reakcje



$$\sum X : H_E - 10 \cos 60^\circ = 0$$

$$\sum Y : R_A + R_B + V_E - 8kN/m \cdot 3,5m - 10kN \sin 60^\circ = 0$$

$$\sum M_E : R_A \cdot 9m + R_B \cdot 6,5m + M_E +$$

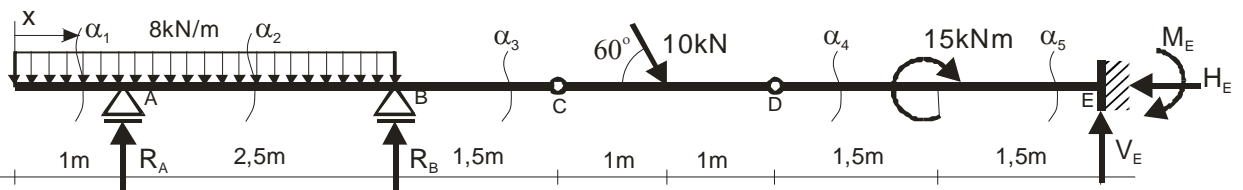
$$+ 15kNm - 10kN \sin 60^\circ \cdot 4m - 8 \cdot 3,5m \cdot \left(\frac{1}{2} 3,5m + 6,5m \right) = 0$$

$$\sum M_C^l : R_A \cdot 4m + R_B \cdot 1,5m - 8kN/m \cdot 3,5m \cdot \left(\frac{1}{2} 3,5m + 1,5m \right) = 0$$

$$\sum M_D^l : R_A \cdot 6m + R_B \cdot 3,5m - 8kN/m \cdot 3,5m \cdot \left(\frac{1}{2} 3,5m + 3,5m \right) - 10kN \sin 60^\circ \cdot 1m = 0$$

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Przyjęcie przekrojów, przedziały



$$R_A = 17,002kN$$

$$R_B = 15,328kN$$

$$H_E = 5kN$$

$$V_E = 4,330kN$$

$$M_E = -2,010kN$$

$$\alpha_1 - \alpha_1 \quad x \in \langle 0; 1m \rangle$$

$$\alpha_2 - \alpha_2 \quad x \in \langle 1m; 3,5m \rangle$$

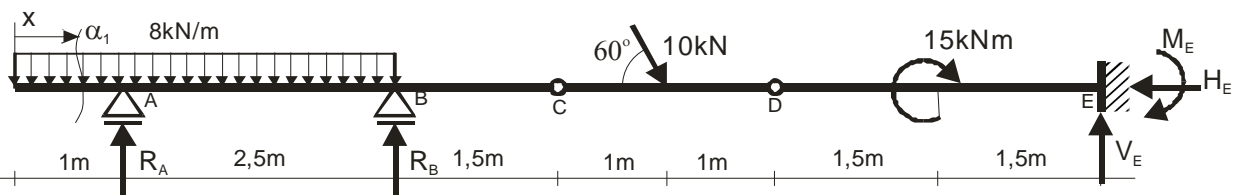
$$\alpha_3 - \alpha_3 \quad x \in \langle 3,5m; 6m \rangle$$

$$\alpha_4 - \alpha_4 \quad x \in \langle 6m; 8,5m \rangle$$

$$\alpha_5 - \alpha_5 \quad x \in \langle 8,5m; 10m \rangle$$

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Przekrój $\alpha_1 - \alpha_1$ $x \in \langle 0; 1m \rangle$



$$N_{\alpha_1} = 0$$

$$T_{\alpha_1} = -8kN / m \cdot x$$

$$x = 0 \quad T_{\alpha_1} = 0$$

$$x = 1m \quad T_{\alpha_1} = -8kN$$

$$M_{\alpha_1} = -8kN / m \cdot x \cdot \frac{x}{2} =$$

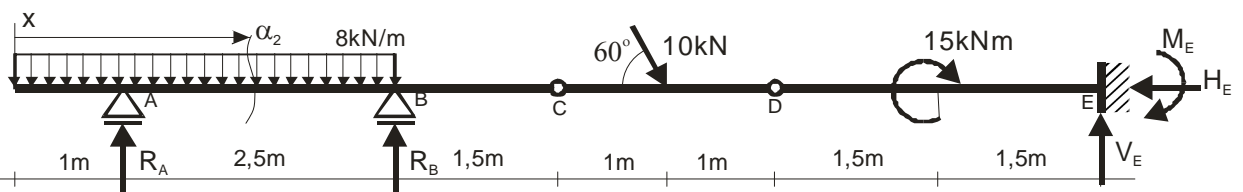
$$= -4kN / m \cdot x^2$$

$$x = 0 \quad M_{\alpha_1} = 0$$

$$x = 1m \quad M_{\alpha_1} = -4kNm$$

5

Przekrój $\alpha_2 - \alpha_2$ $x \in \langle 1m; 3,5m \rangle$



$$N_{\alpha_2} = 0$$

$$T_{\alpha_2} = -8kN / m \cdot x + 17,002kN$$

$$x = 1m \quad T_{\alpha_2} = 9,002kN$$

$$x = 3,5m \quad T_{\alpha_2} = -10,998kN$$

$$M_{\alpha_2} = -8kN / m \cdot x \cdot \frac{x}{2} + 17,002kN \cdot (x - 1m) =$$

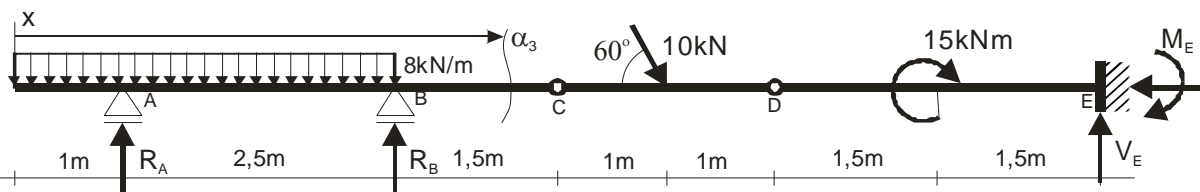
$$= -4kN / m \cdot x^2 + 17,002kN \cdot x - 17,002kNm$$

$$x = 1m \quad M_{\alpha_2} = -4kNm$$

$$x = 3,5m \quad M_{\alpha_2} = -6,495kNm$$

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Przekrój $\alpha_3 - \alpha_3$ $x \in \langle 3,5m; 6m \rangle$



$$N_{\alpha_3} = 0$$

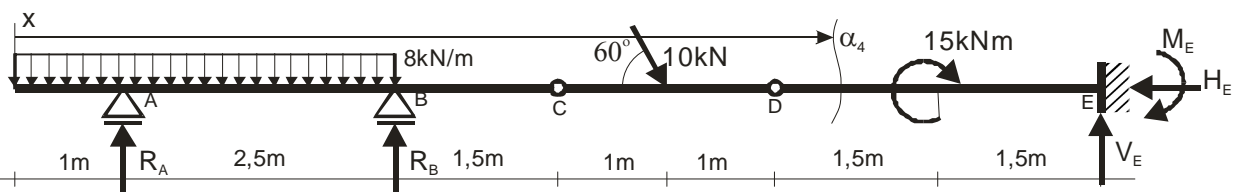
$$T_{\alpha_3} = -8kN/m \cdot 3,5m + 17,002kN + 15,328kN \\ = 4,33kN$$

$$M_{\alpha_3} = -8kN/m \cdot 3,5m \cdot \left(x - \frac{3,5m}{2}\right) + \\ + 17,002kN \cdot (x - 1m) + 15,328kN \cdot (x - 3,5m) = \begin{cases} x = 3,5m & M_{\alpha_3} = -6,495kNm \\ x = 5m & M_{\alpha_3} = 0 \\ x = 6m & M_{\alpha_3} = 4,330kNm \end{cases}$$

$$= 4,33kN \cdot x - 21,65kNm$$

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Przekrój $\alpha_4 - \alpha_4$ $x \in \langle 6m; 8,5m \rangle$



$$N_{\alpha_4} = -10kN \cos 60^\circ = -5kN$$

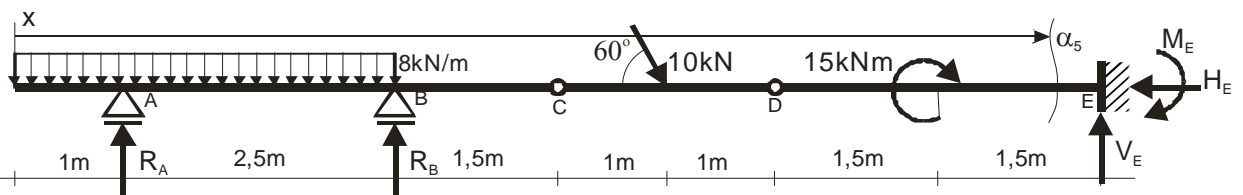
$$T_{\alpha_4} = 4,33kN - 10kN \sin 60^\circ = -4,33kN$$

$$M_{\alpha_4} = 4,33kN \cdot x - 21,65kNm - 10kN \sin 60^\circ \cdot (x - 6m) = \\ = -4,33kN \cdot x + 30,31kNm$$

$$\begin{cases} x = 6m & M_{\alpha_4} = 4,33m \\ x = 7m & M_{\alpha_4} = 0 \\ x = 8,5m & M_{\alpha_4} = -6,495kNm \end{cases}$$

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Przekrój $\alpha_5 - \alpha_5 \quad x \in \langle 8,5m; 10m \rangle$



$$N_{\alpha_5} = -5 \text{ kN}$$

$$T_{\alpha_5} = -4,33 \text{ kN}$$

$$M_{\alpha_5} = -4,33 \text{ kN} \cdot x + 30,31 \text{ kNm} + 15 \text{ kNm} =$$

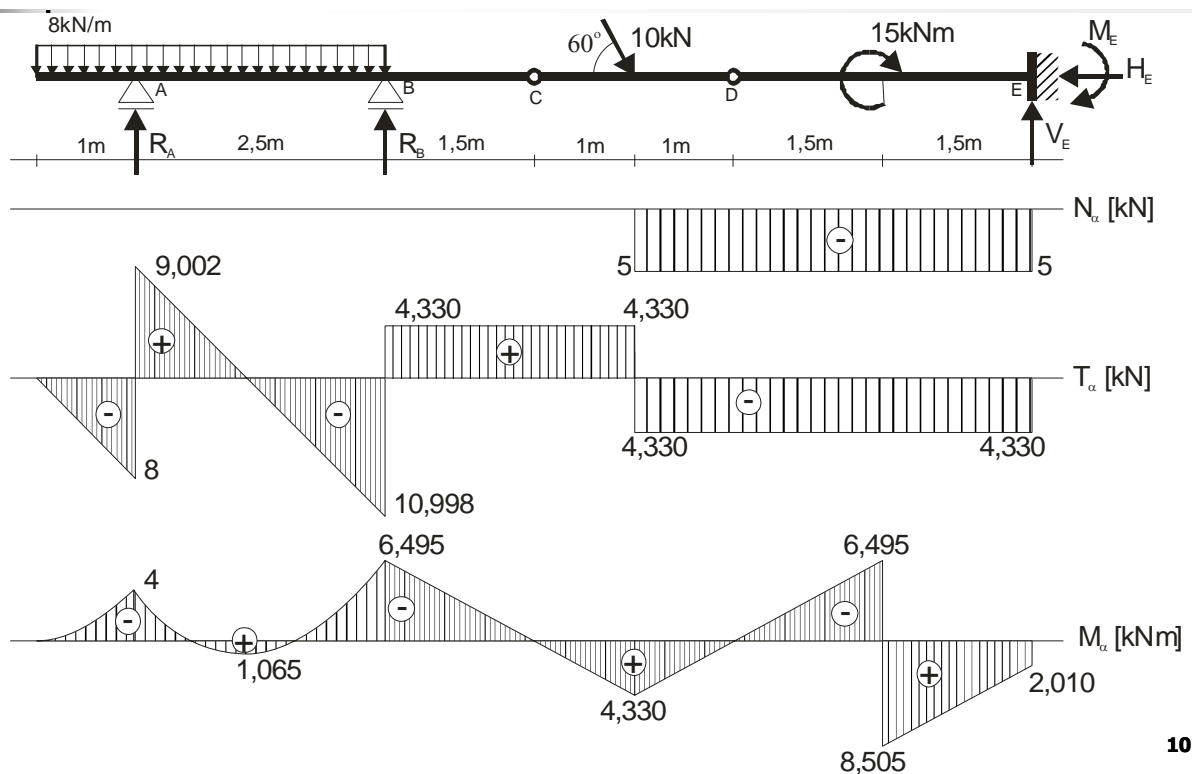
$$= -4,33 \text{ kN} \cdot x + 45,31 \text{ kNm}$$

$$x = 8,5 \text{ m} \quad M_{\alpha_5} = 8,505 \text{ kNm}$$

$$x = 10 \text{ m} \quad M_{\alpha_5} = 2,01 \text{ kNm}$$

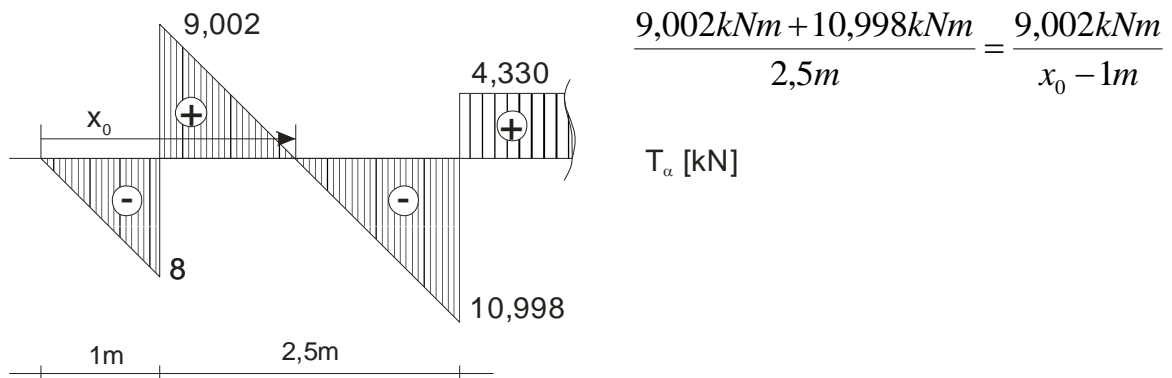
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Wykresy



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Ekstremum



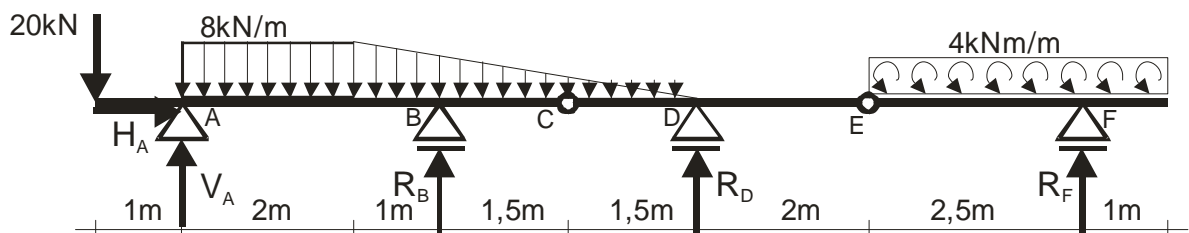
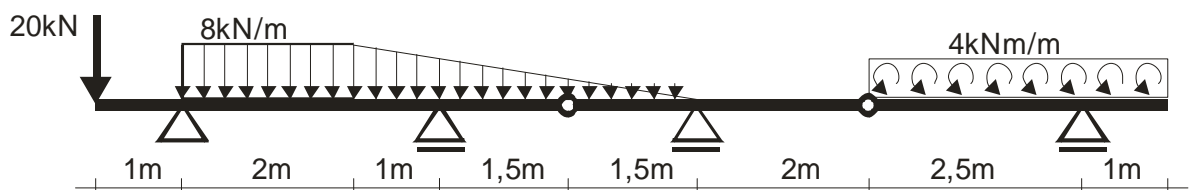
$$T_{\alpha 2} = -8kN/m \cdot x + 17,002kN = 0 \qquad x_0 = 2,125m$$

$$M_{\alpha 2} = -4kN/m \cdot x^2 + 17,002kN \cdot x - 17,002kNm$$

$$M_{\alpha 2}(x_0 = 2,125m) = 1,065kNm$$

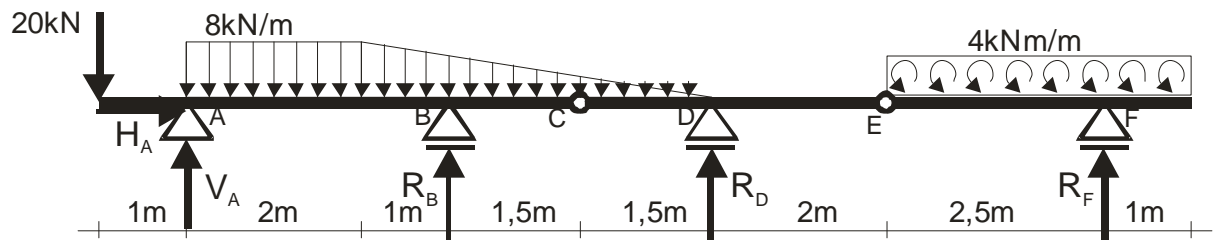
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Przykład – belka przegubowa



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Reakcje



$$\sum X : H_A = 0$$

$$\sum Y : V_A + R_B + R_D + R_F - 20kN - 8kN/m \cdot 2m - \frac{1}{2} 8kN/m \cdot 4m = 0$$

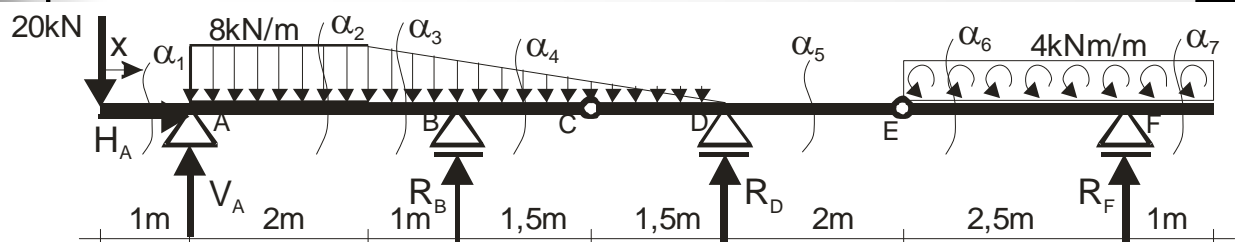
$$\sum M_A : R_B \cdot 3m + R_D \cdot 6m + R_F \cdot 10,5m + 20kN \cdot 1m - 8kN/m \cdot 2m \cdot 1m + \\ - \frac{1}{2} 8kN/m \cdot 4m \cdot \left(2m + \frac{1}{3} 4m \right) + 4kNm/m \cdot 3,5m = 0$$

$$\sum M_E^P : R_F \cdot 2,5m + 4kNm/m \cdot 3,5m = 0$$

$$\sum M_C^P : R_D \cdot 1,5m + R_F \cdot 6m - \frac{1}{2} 3kN/m \cdot 1,5m \cdot \frac{1}{3} 1,5m + 4kNm/m \cdot 3,5m = 0$$

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Przyjęcie przekrojów, przedziały



$$H_A = 0$$

$$R_F = 5,6kN$$

$$R_D = 13,817kN$$

$$R_B = 3,744kN$$

$$V_A = 40,039kN$$

$$\alpha_1 - \alpha_1 \quad x \in \langle 0; 1m \rangle$$

$$\alpha_2 - \alpha_2 \quad x \in \langle 1m; 3m \rangle$$

$$\alpha_3 - \alpha_3 \quad x \in \langle 3m; 4m \rangle$$

$$\alpha_4 - \alpha_4 \quad x \in \langle 4m; 7m \rangle$$

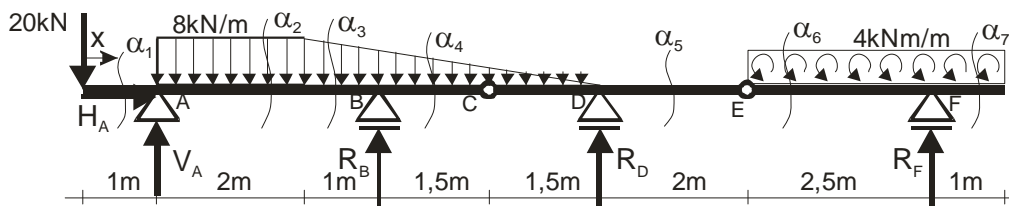
$$\alpha_5 - \alpha_5 \quad x \in \langle 7m; 9m \rangle$$

$$\alpha_6 - \alpha_6 \quad x \in \langle 9m; 11,5m \rangle$$

$$\alpha_7 - \alpha_7 \quad x \in \langle 11,5m; 12,5m \rangle$$

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Przekrój $\alpha_1 - \alpha_1$ $x \in \langle 0; 1m \rangle$



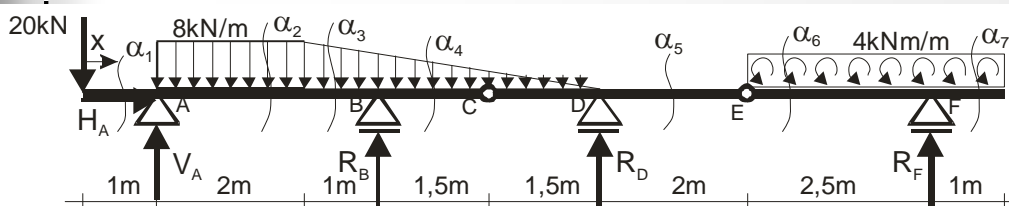
$$N_{\alpha_1} = 0$$

$$T_{\alpha_1} = -20kN$$

$$M_{\alpha_1} = -20kN \cdot x \quad \left| \begin{array}{l} x = 0 \quad M_{\alpha_1} = 0 \\ x = 1m \quad M_{\alpha_1} = -20kNm \end{array} \right.$$

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Przekrój $\alpha_2 - \alpha_2$ $x \in \langle 1m; 3m \rangle$



$$N_{\alpha_2} = H_A = 0$$

$$T_{\alpha_2} = -20kN + V_A - 8kN/m \cdot (x-1m) = -20kN + 40,039kN - 8kN/m \cdot (x-1m) = 28,039kN - 8kN/m \cdot x$$

$$\left| \begin{array}{l} x = 1m \quad T_{\alpha_2} = 20,039kN \\ x = 3m \quad T_{\alpha_2} = 4,039kN \end{array} \right.$$

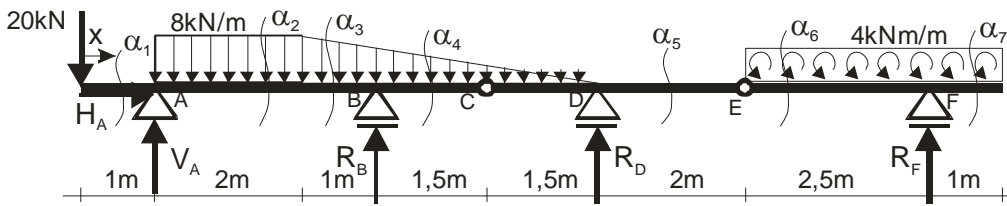
$$M_{\alpha_2} = -20kN \cdot x + V_A \cdot (x-1m) - 8kN/m \cdot (x-1m) \cdot \frac{x-1m}{2} =$$

$$= -20kN \cdot x + 40,039kN \cdot (x-1m) - 8kN/m \cdot (x-1m) \cdot \frac{x-1m}{2} =$$

$$= -4kN/m \cdot x^2 + 28,039kN \cdot x - 44,039kNm$$

$$\left| \begin{array}{l} x = 1m \quad M_{\alpha_2} = -20kNm \\ x = 3m \quad M_{\alpha_2} = 4,078kNm \end{array} \right. \quad 16$$

Przekrój $\alpha_3 - \alpha_3 \quad x \in \langle 3m; 4m \rangle$ – siły normalne i tnące



$$\frac{q_3(x)}{7m-x} = \frac{8kN/m}{4m}$$

$$N_{\alpha_3} = H_A = 0$$

$$q_3(x) = 2kN/m^2 \cdot (7m-x)$$

$$T_{\alpha_3} = -20kN + V_A - 8kN/m \cdot 2m - q_3(x) \cdot (x-3m) - \frac{1}{2}(8kN/m - q_3(x)) \cdot (x-3m) =$$

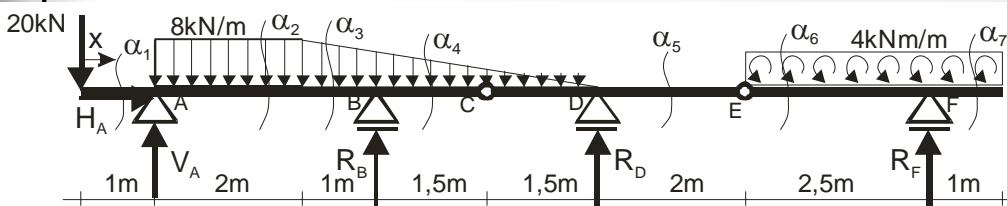
$$= -20kN + 40,039kN - 16kN - 2kN/m^2 \cdot (7m-x) \cdot (x-3m) +$$

$$- \frac{1}{2}(8kN/m - 2kN/m^2 \cdot (7m-x)) \cdot (x-3m) = \begin{cases} x=3m & T_{\alpha_3} = 4,039kN \\ x=4m & T_{\alpha_3} = -2,961kN \end{cases}$$

$$= 1 \frac{kN}{m^2} x^2 - 14 \frac{kN}{m} x + 37,039kN$$

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Przekrój $\alpha_3 - \alpha_3 \quad x \in \langle 3m; 4m \rangle$ – momenty zginające



$$M_{\alpha_3} = -20kN \cdot x + V_A \cdot (x-1m) - 8kN/m \cdot 2m \cdot (x-2m) +$$

$$- q_3(x) \cdot (x-3m) \frac{(x-3m)}{2} - \frac{1}{2}(8kN/m - q_3(x)) \cdot (x-3m) \cdot \frac{2}{3}(x-3m) =$$

$$= -20kN \cdot x + 40,039kN \cdot (x-1m) - 16kN \cdot (x-2m) +$$

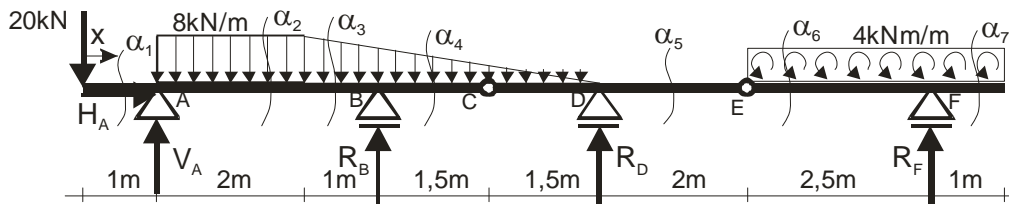
$$- 2kN/m^2 \cdot (7m-x) \cdot (x-3m) \frac{(x-3m)}{2} +$$

$$- \frac{1}{3}(8kN/m - 2kN/m^2 \cdot (7m-x)) \cdot (x-3m) = \begin{cases} x=3m & M_{\alpha_3} = 4,078kNm \\ x=4m & M_{\alpha_3} = 4,450kNm \end{cases}$$

$$= \frac{1}{3} kN/m^2 \cdot x^3 - 7kN/m \cdot x^2 + 37,039kN \cdot x - 53,039kNm$$

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Przekrój $\alpha_4 - \alpha_4$ $x \in \langle 4m; 7m \rangle$



$$N_{\alpha_4} = H_A = 0$$

$$T_{\alpha_4} = -20kN + V_A - 8kN/m \cdot 2m - q_3(x) \cdot (x-3m) - \frac{1}{2}(8kN/m - q_3(x)) \cdot (x-3m) + R_B =$$

$$= 1 \frac{kN}{m^2} x^2 - 14 \frac{kN}{m} x + 37,039kN + 3,744kN =$$

$$= 1 \frac{kN}{m^2} x^2 - 14 \frac{kN}{m} x + 40,783kN$$

$$x = 4m \quad T_{\alpha_4} = 0,783kN$$

$$x = 7m \quad T_{\alpha_4} = -8,217kN$$

$$M_{\alpha_4} = -20kN \cdot x + V_A \cdot (x-1m) - 8kN/m \cdot 2m \cdot (x-2m) - q_3(x) \cdot (x-3m) \frac{(x-3m)}{2} +$$

$$- \frac{1}{2}(8kN/m - q_3(x)) \cdot (x-3m) \cdot \frac{2}{3}(x-3m) + R_B \cdot (x-4m) =$$

$$= \frac{1}{3} kN/m^2 \cdot x^3 - 7kN/m \cdot x^2 + 40,783kN \cdot x - 68,015kNm$$

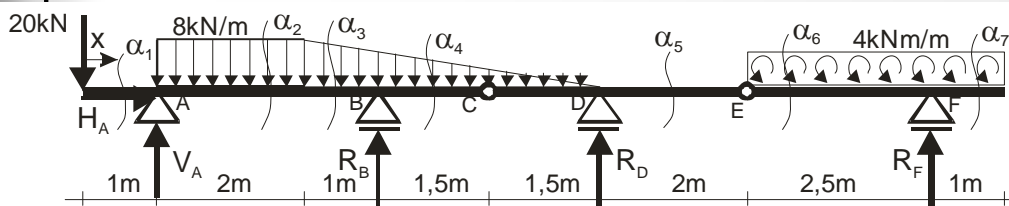
$$x = 4m \quad M_{\alpha_4} = 4,450kNm$$

$$x = 5,5m \quad M_{\alpha_4} = 0$$

$$x = 7m \quad M_{\alpha_4} = -11,201kNm$$

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Przekrój $\alpha_5 - \alpha_5$ $x \in \langle 7m; 9m \rangle$



$$N_{\alpha_5} = H_A = 0$$

$$T_{\alpha_5} = -20kN + V_A - 8kN/m \cdot 2m - \frac{1}{2} 8kN/m \cdot 4m + R_B + R_D =$$

$$= -20kN + 40,039kN - 16kN - 16kN + 3,744kN + 13,817kN =$$

$$= 5,6kN$$

$$M_{\alpha_5} = -20kN \cdot x + V_A \cdot (x-1m) - 8kN/m \cdot 2m \cdot (x-2m) +$$

$$- \frac{1}{2} 8kN/m \cdot 4m \cdot \left(x - \left(3m + \frac{1}{3} 4m \right) \right) + R_B \cdot (x-4m) + R_D \cdot (x-7m) =$$

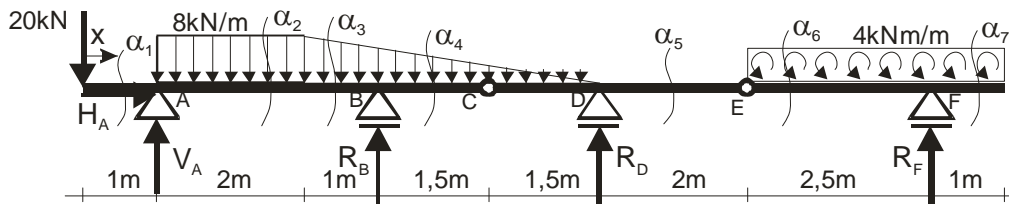
$$= 5,6kN \cdot x - 50,401kNm$$

$$x = 7m \quad M_{\alpha_5} = -11,201kNm$$

$$x = 9m \quad M_{\alpha_5} = -0,001kNm \approx 0$$

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Przekrój $\alpha_6 - \alpha_6$ $x \in \langle 9m; 11,5m \rangle$



$$N_{\alpha_6} = H_A = 0$$

$$T_{\alpha_6} = 5,6kN$$

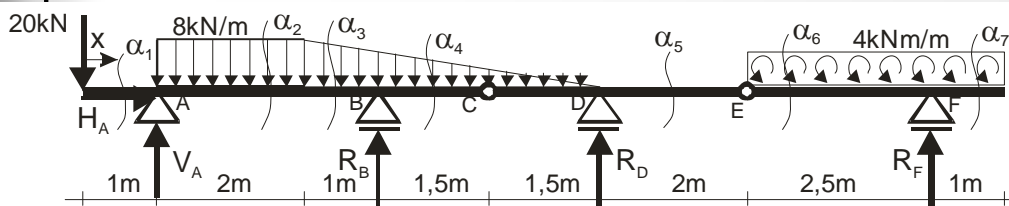
$$M_{\alpha_6} = 5,6kN \cdot x - 50,401kNm - 4kNm/m \cdot (x - 9m) =$$

$$= 1,6kN \cdot x - 14,401kNm$$

$$\left| \begin{array}{l} x = 9m \quad M_{\alpha_6} = -0,001kNm \approx 0 \\ x = 11,5m \quad M_{\alpha_6} = 3,999kNm \end{array} \right.$$

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Przekrój $\alpha_7 - \alpha_7$ $x \in \langle 11,5m; 12,5m \rangle$



$$N_{\alpha_7} = H_A = 0$$

$$T_{\alpha_7} = 5,6kN - R_F = 0$$

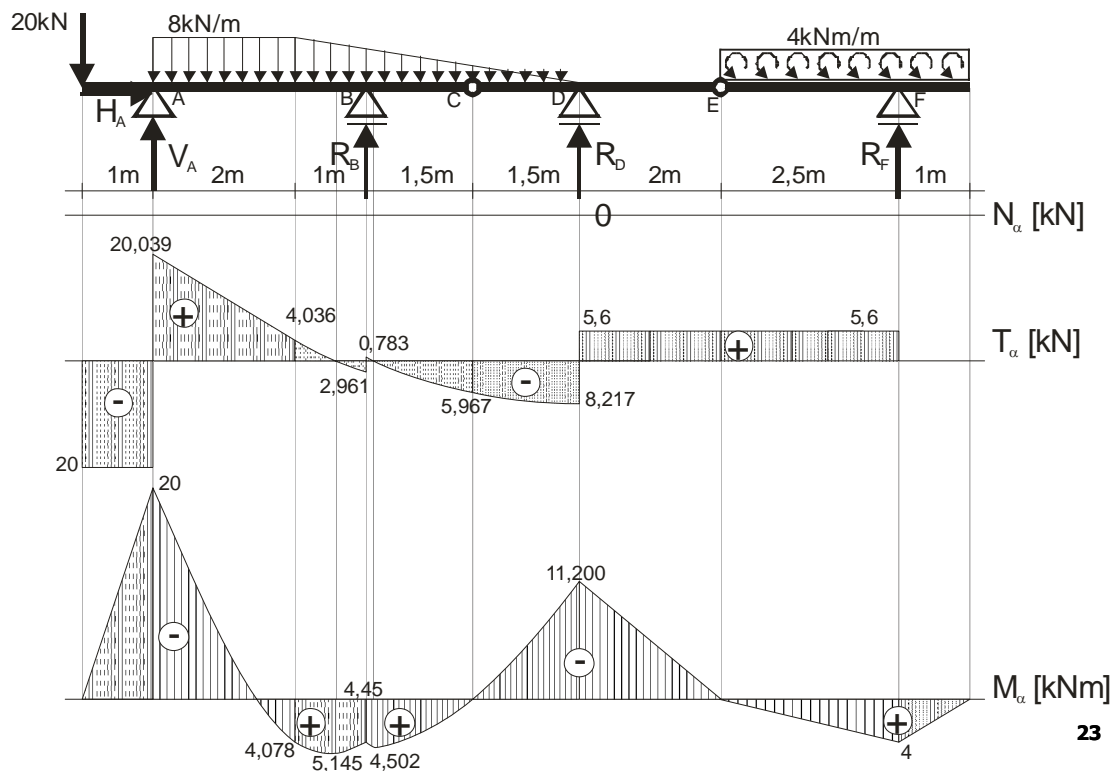
$$M_{\alpha_7} = 1,6kN \cdot x - 14,401kNm - 5,6kN \cdot (x - 11,5m) =$$

$$= -4kN \cdot x + 49,999kNm$$

$$\left| \begin{array}{l} x = 11,5m \quad M_{\alpha_7} = 3,999kNm \\ x = 12,5m \quad M_{\alpha_7} = -0,001kNm \approx 0 \end{array} \right.$$

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Wykresy



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Ekstrema

$$T_{\alpha 3} = 1 \frac{kN}{m^2} x^2 - 14 \frac{kN}{m} x + 37,039 kN = 0 \quad x_0 = 3,542 m$$

$$M_{\alpha 3}(x_0) = \frac{1}{3} kN / m^2 \cdot x_0^3 - 7 kN / m \cdot x_0^2 + 37,039 kN \cdot x_0 - 53,039 kNm =$$

$$= 5,145 kNm$$

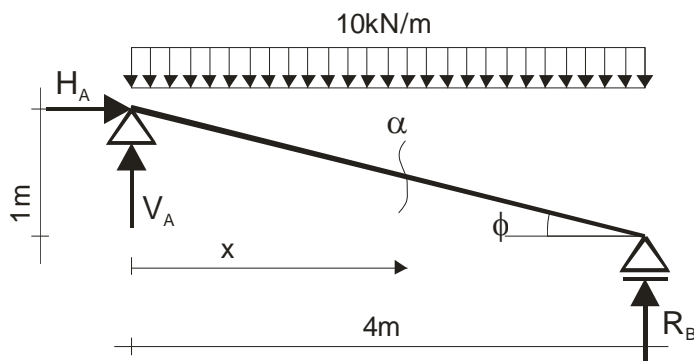
$$T_{\alpha 4} = 1 \frac{kN}{m^2} x^2 - 14 \frac{kN}{m} x + 40,783 kN = 0 \quad x_0 = 4,133 m$$

$$M_{\alpha 4}(x_0) = \frac{1}{3} kN / m^2 \cdot x_0^3 - 7 kN / m \cdot x_0^2 + 40,783 kN \cdot x_0 - 68,015 kNm =$$

$$= 4,502 kNm$$

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Obciążenie na pręcie ukośnym – na jednostkę rzutu



$$\sin \phi = \frac{1m}{\sqrt{(4m)^2 + (1m)^2}} = 0,243$$

$$\cos \phi = \frac{4m}{\sqrt{(4m)^2 + (1m)^2}} = 0,970$$

$$\sum X : H_A = 0$$

$$H_A = 0$$

$$\sum Y : V_A + R_B - 10 \frac{kN}{m} \cdot 4m = 0$$

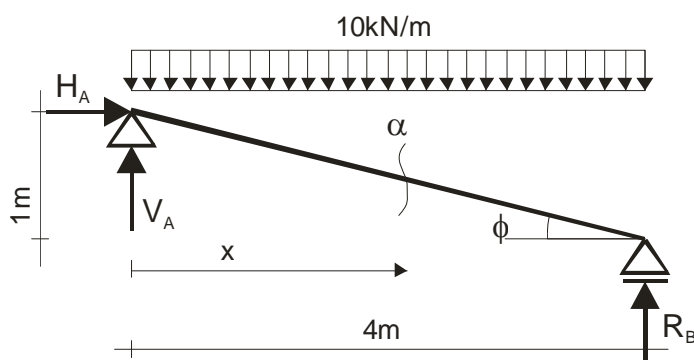
$$V_A = 20kN$$

$$\sum M_A : R_B \cdot 4m - 10 \frac{kN}{m} \cdot 4m \cdot \frac{1}{2} 4m = 0$$

$$R_B = 20kN$$

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Siły wewnętrzne



$$N_\alpha = V_A \cdot \sin \phi - q \cdot x \cdot \sin \phi =$$

$$= 4,851kN - 2,425 \frac{kN}{m} \cdot x$$

$$T_\alpha = V_A \cdot \cos \phi - q \cdot x \cdot \cos \phi =$$

$$= 19,403kN - 9,701 \frac{kN}{m} \cdot x$$

$$M_\alpha = V_A \cdot x - 10 \frac{kN}{m} \cdot x \cdot \frac{x}{2} =$$

$$= -5 \frac{kN}{m} \cdot x^2 + 20kN \cdot x$$

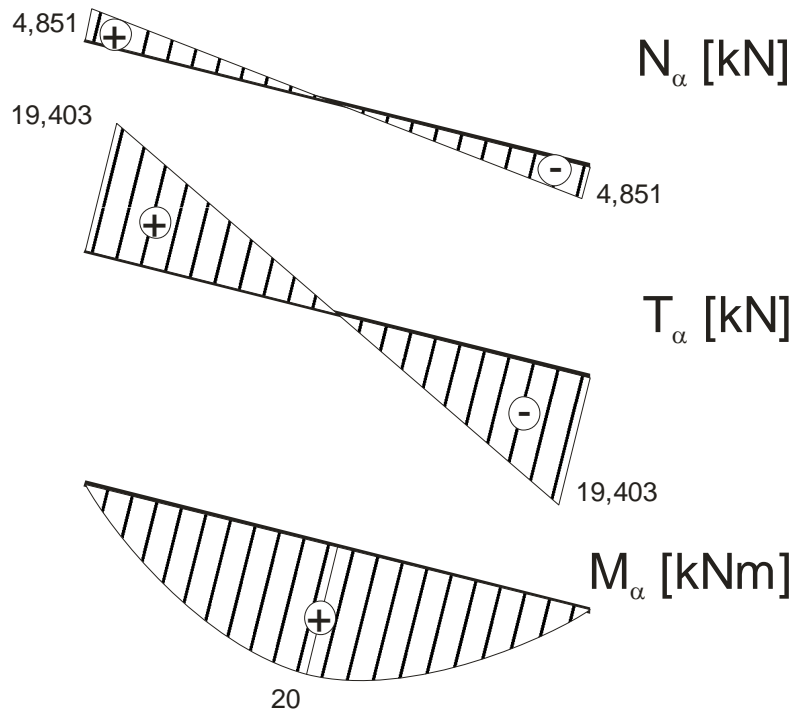
$$x = 0 \quad N_\alpha = 4,851kN \quad T_\alpha = 19,403kN \quad M_\alpha = 0$$

$$x = 2m \quad N_\alpha = 0 \quad T_\alpha = 0 \quad M_\alpha = 20kNm$$

$$x = 4m \quad N_\alpha = -4,849kN \quad T_\alpha = -19,401kN \quad M_\alpha = 0$$

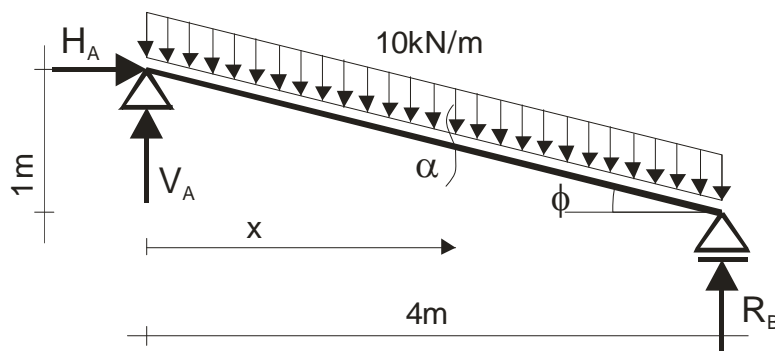
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Wykresy



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Obciążenie na pręcie ukośnym – na jednostkę długości pręta



$$\sum X : H_A = 0$$

$$\sum Y : V_A + R_B - 10 \frac{\text{kN}}{\text{m}} \cdot \sqrt{(4\text{m})^2 + (1\text{m})^2} = 0$$

$$\sum M_A : R_B \cdot 4\text{m} - 10 \frac{\text{kN}}{\text{m}} \cdot \sqrt{(4\text{m})^2 + (1\text{m})^2} \cdot \frac{1}{2} 4\text{m} = 0$$

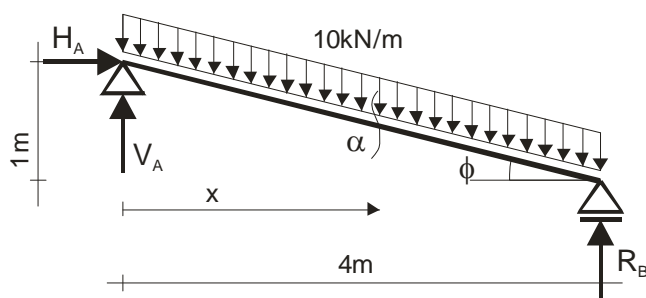
$$H_A = 0$$

$$V_A = 20,616\text{ kN}$$

$$R_B = 20,616\text{ kN}$$

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Siły wewnętrzne



$$N_{\alpha} = V_A \cdot \sin \phi - q \cdot \sqrt{x^2 + \left(\frac{x}{4}\right)^2} \cdot \sin \phi =$$

$$= 5 \text{ kN} - 2,5 \frac{\text{kN}}{\text{m}} \cdot x$$

$$T_{\alpha} = V_A \cdot \cos \phi - q \cdot \sqrt{x^2 + \left(\frac{x}{4}\right)^2} \cdot \cos \phi =$$

$$= 20 \text{ kN} - 10 \frac{\text{kN}}{\text{m}} \cdot x$$

$$M_{\alpha} = V_A \cdot x - 10 \frac{\text{kN}}{\text{m}} \cdot \sqrt{x^2 + \left(\frac{x}{4}\right)^2} \cdot \frac{x}{2} = -5,154 \frac{\text{kN}}{\text{m}} \cdot x^2 + 20,616 \text{ kN} \cdot x$$

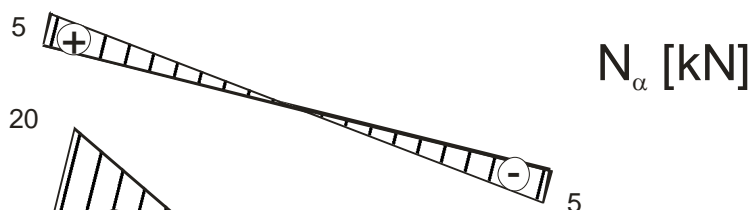
$$x = 0 \quad N_{\alpha} = 5 \text{ kN} \quad T_{\alpha} = 20 \text{ kN} \quad M_{\alpha} = 0$$

$$x = 2 \text{ m} \quad N_{\alpha} = 0 \quad T_{\alpha} = 0 \quad M_{\alpha} = 20,616 \text{ kNm}$$

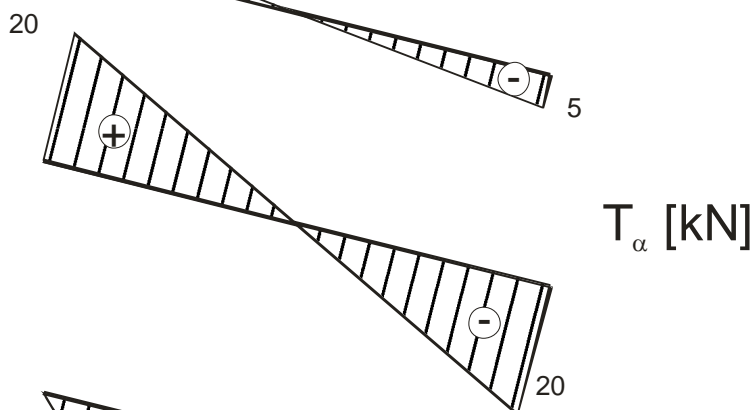
$$x = 4 \text{ m} \quad N_{\alpha} = -5 \quad T_{\alpha} = -20 \text{ kN} \quad M_{\alpha} = 0$$

29

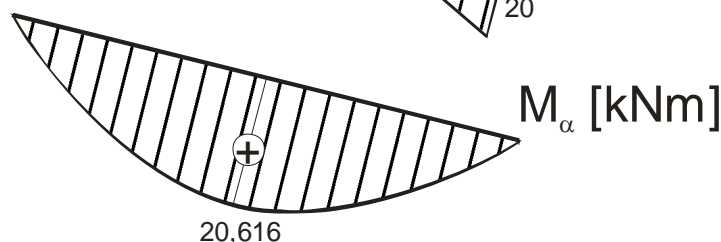
Wykresy



N_{α} [kN]



T_{α} [kN]



M_{α} [kNm]

20,616

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