

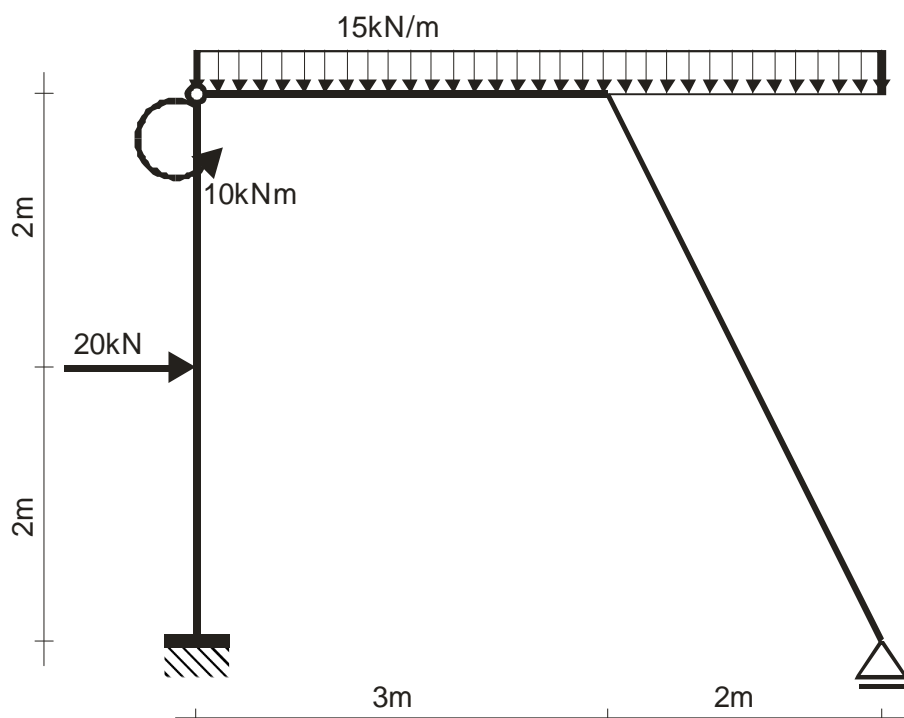
Mechanika ogólna

Wykład nr 7

Obliczanie sił wewnętrznych w ramach.

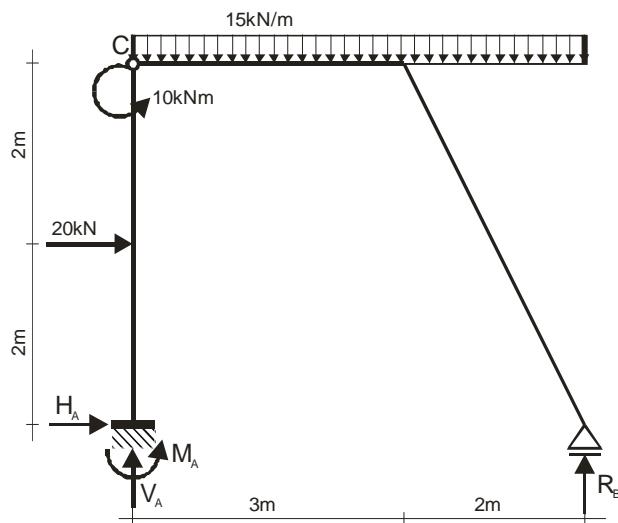
1

Przykład – rama



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Reakcje



$$\sum X : H_A + 20kN = 0$$

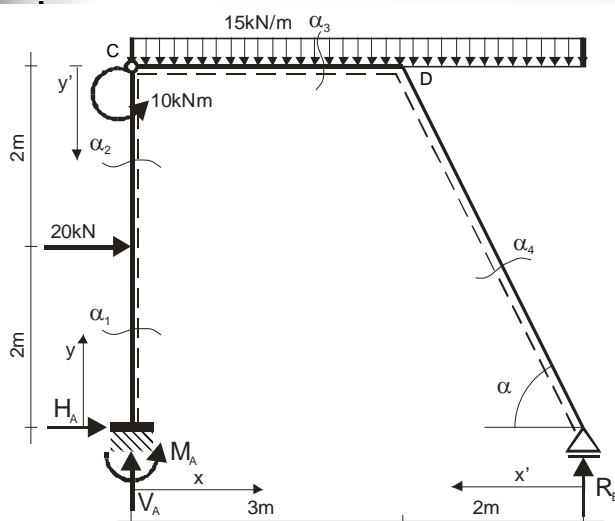
$$\sum Y : V_A + R_B - 15kN \cdot 5m = 0$$

$$\sum M_A : M_A + R_B \cdot 5m + 10kNm - 20kN \cdot 2m - 15kN/m \cdot 5m \cdot \frac{1}{2} \cdot 5m = 0$$

$$\sum M_C^P : R_B \cdot 5m - 15kN/m \cdot 5m \cdot \frac{1}{2} \cdot 5m = 0$$

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Przyjęcie przekrojów, przedziały



$$H_A = -20kN$$

$$V_A = 37,5kN$$

$$R_B = 37,5kN$$

$$M_A = 30kNm$$

$$\alpha_1 - \alpha_1 \quad y \in \langle 0; 2m \rangle$$

$$\alpha_2 - \alpha_2 \quad y \in \langle 2m; 4m \rangle$$

$$\alpha_3 - \alpha_3 \quad x \in \langle 0m; 3m \rangle$$

$$\alpha_4 - \alpha_4 \quad x \in \langle 3m; 5m \rangle \quad y \in \langle 0m; 4m \rangle$$

$$x' \in \langle 0m; 2m \rangle \quad y' \in \langle 0m; 4m \rangle$$

$$\text{punkt D} \quad x = 3m, \quad y = 4m, \quad x' = 2m, \quad y' = 0$$

$$\text{punkt B} \quad x = 5m, \quad y = 0, \quad x' = 0, \quad y' = 4m$$

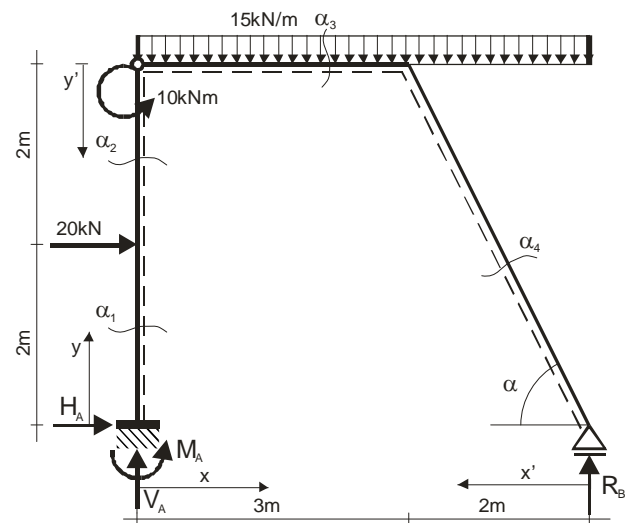
4

Przekrój $\alpha_1 - \alpha_1$ $y \in \langle 0; 2m \rangle$

$$N_{\alpha_1} = -V_A = -37,5kN$$

$$T_{\alpha_1} = -H_A = 20kN$$

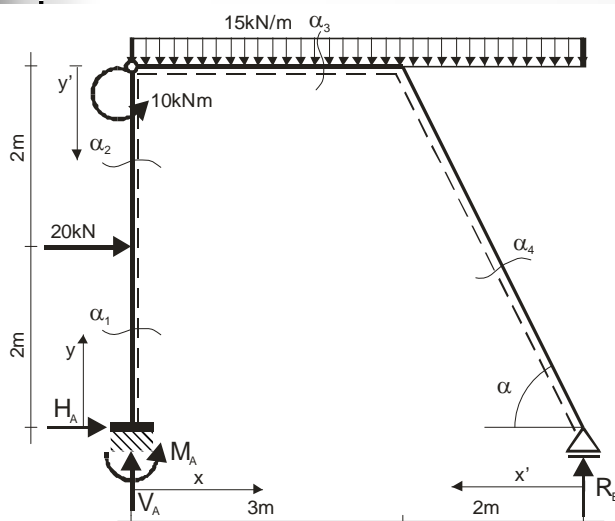
$$M_{\alpha_1} = -M_A - H_A \cdot y = \\ = 20kN \cdot y - 30kNm$$



$$\left| \begin{array}{l} y = 0 \quad M_{\alpha_1} = -30kNm \\ y = 2m \quad M_{\alpha_1} = 10kNm \end{array} \right.$$

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Przekrój $\alpha_2 - \alpha_2$ $y \in \langle 2m; 4m \rangle$



$$N_{\alpha_2} = -37,5kN$$

$$T_{\alpha_2} = -H_A - 20kN = 0$$

$$M_{\alpha_2} = 20kN \cdot y - 30kNm - 20kN \cdot (y - 2m) = 10kNm$$

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Przekrój $\alpha_3 - \alpha_3 \quad x \in \langle 0; 3m \rangle$

$$N_{\alpha_3} = -H_A - 20kN = 20kN - 20kN = 0$$

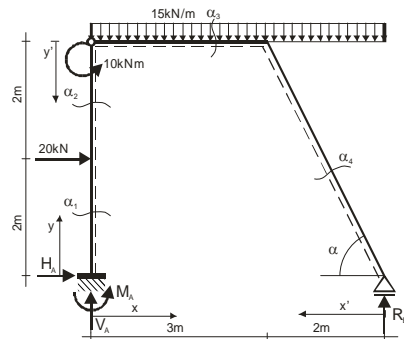
$$T_{\alpha_3} = V_A - 15kN/m \cdot x = 37,5kN - 15kN/m \cdot x$$

$$\left| \begin{array}{l} x = 0m \quad T_{\alpha_3} = 37,5kN \\ x = 3m \quad T_{\alpha_3} = -7,5kN \end{array} \right.$$

$$M_{\alpha_3} = V_A \cdot x - H_A \cdot 4m - M_A - 20kN \cdot 2m - 10kNm - 15kN/m \cdot x \cdot \frac{x}{2} =$$

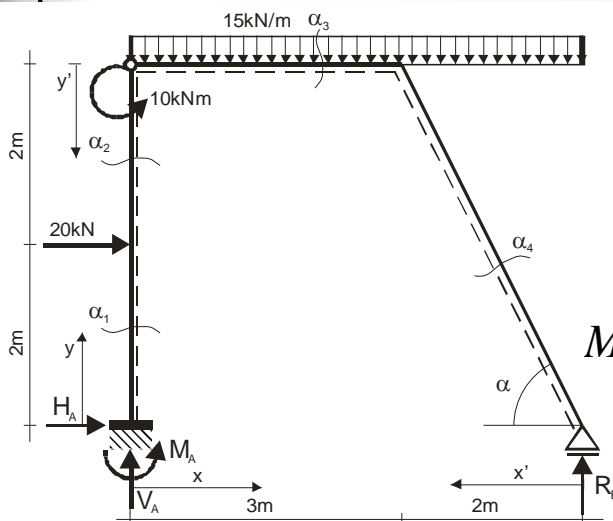
$$= -7,5kN/m \cdot x^2 + 37,5kN \cdot x$$

$$\left| \begin{array}{l} x = 0m \quad M_{\alpha_3} = 0 \\ x = 3m \quad M_{\alpha_3} = 45kNm \end{array} \right.$$



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Ekstremum



$$T_{\alpha_3} = 37,5kN - 15kN/m \cdot x = 0$$

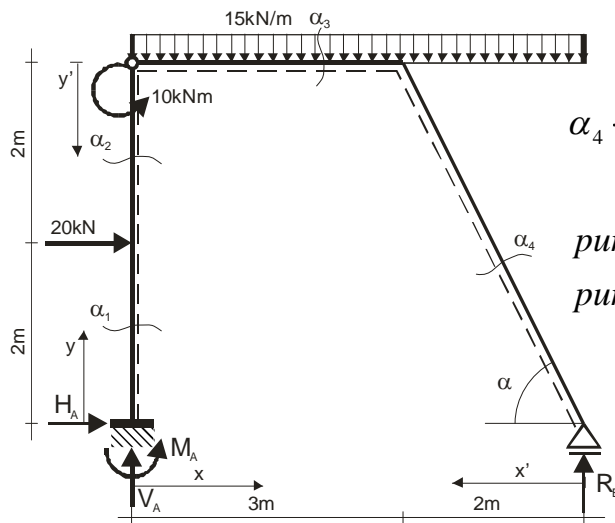
$$x = 2,5m$$

$$M_{\alpha_3} = -7,5kN/m \cdot x^2 + 37,5kN \cdot x$$

$$M_{\alpha_3}(2,5m) = 46,875kNm$$

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Przekrój $\alpha_4 - \alpha_4$



$$\alpha_4 - \alpha_4 \quad x \in \langle 3m; 5m \rangle \quad y \in \langle 0m; 4m \rangle$$

$$x' \in \langle 0m; 2m \rangle \quad y' \in \langle 0m; 4m \rangle$$

$$\text{punkt } D \quad x = 3m, \quad y = 4m, \quad x' = 2m, \quad y' = 0$$

$$\text{punkt } B \quad x = 5m, \quad y = 0, \quad x' = 0, \quad y' = 4m$$

$$\sin \alpha = \frac{4m}{\sqrt{(4m)^2 + (2m)^2}} = 0,894$$

$$\cos \alpha = \frac{2m}{\sqrt{(4m)^2 + (2m)^2}} = 0,447$$

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Przekrój $\alpha_4 - \alpha_4$ (z lewej strony)

$$N_{\alpha_4} = -H_A \cos \alpha - 20kN \cos \alpha + V_A \sin \alpha - q \cdot x \sin \alpha =$$

$$= 37,5kN \sin \alpha - 15 \frac{kN}{m} x \sin \alpha =$$

$$= -13,41 \frac{kN}{m} x + 33,525kN$$

$$\left| \begin{array}{l} x = 3m \quad N_{\alpha_4} = -6,705kN \\ x = 5m \quad N_{\alpha_4} = -33,525kN \end{array} \right.$$

$$\left| \begin{array}{l} x = 3m \quad N_{\alpha_4} = -6,705kN \\ x = 5m \quad N_{\alpha_4} = -33,525kN \end{array} \right.$$

$$T_{\alpha_4} = H_A \sin \alpha + 20kN \sin \alpha + V_A \cos \alpha - q \cdot x \cos \alpha =$$

$$= 37,5kN \cos \alpha - 15 \frac{kN}{m} x \cos \alpha =$$

$$= -6,705 \frac{kN}{m} x + 16,763kN$$

$$\left| \begin{array}{l} x = 3m \quad N_{\alpha_4} = -3,352kN \\ x = 5m \quad N_{\alpha_4} = -16,762kN \end{array} \right.$$

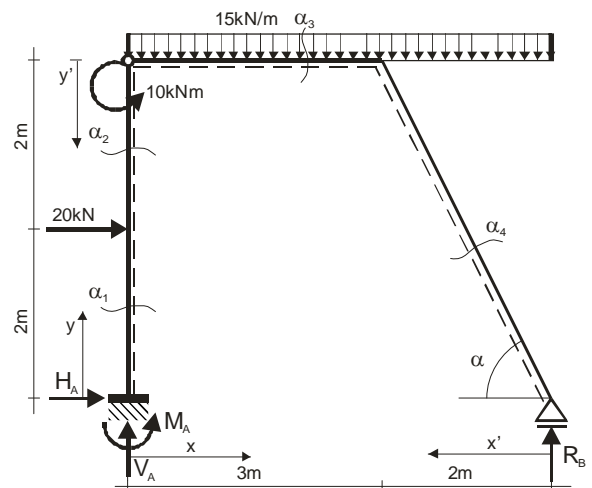
$$\left| \begin{array}{l} x = 3m \quad N_{\alpha_4} = -3,352kN \\ x = 5m \quad N_{\alpha_4} = -16,762kN \end{array} \right.$$

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Przekrój $\alpha_4 - \alpha_4$ (z lewej strony)

$$\begin{aligned}
 M_{\alpha_4} &= V_A \cdot x - H_A \cdot y - 20kN \cdot (y - 2m) - M_A - 15kN/m \cdot x \cdot \frac{x}{2} - 10kNm = \\
 &= 37,5kN \cdot x + 20kN \cdot y - 20kN \cdot y + 40kNm + \\
 &\quad - 30kNm - 7,5 \frac{kN}{m} x^2 - 10kNm = \\
 &= -7,5 \frac{kN}{m} x^2 + 37,5kN \cdot x
 \end{aligned}$$

$$\left| \begin{array}{l} x = 3m \quad M_{\alpha_4} = 45kNm \\ x = 5m \quad M_{\alpha_4} = 0 \end{array} \right.$$



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Przekrój $\alpha_4 - \alpha_4$ (z prawej strony)

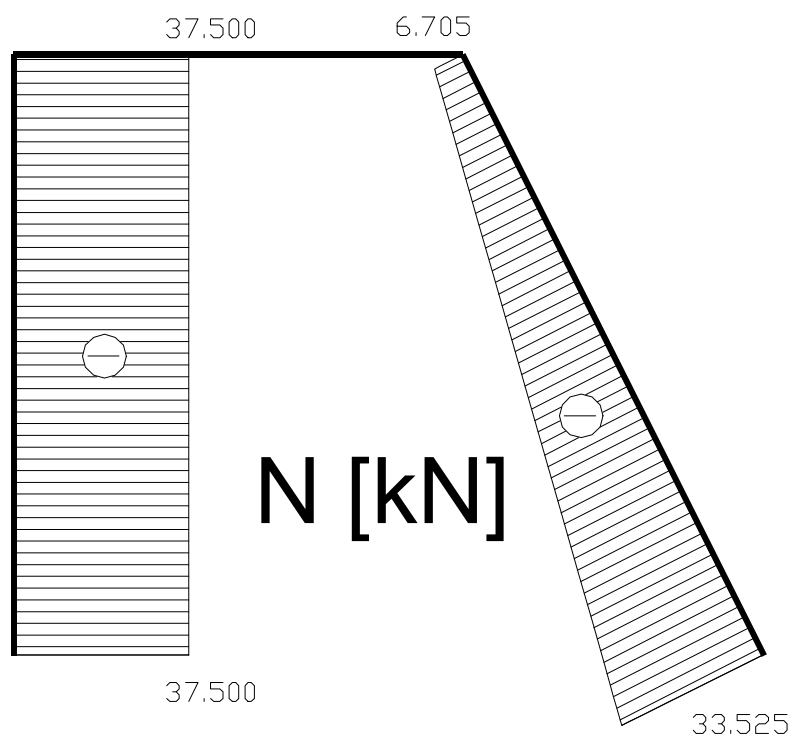
$$\begin{aligned}
 N_{\alpha_4} &= -R_B \sin \alpha + q \cdot x' \sin \alpha = -37,5kN \sin \alpha + 15 \frac{kN}{m} x' \sin \alpha = \\
 &= 13,41 \frac{kN}{m} x' - 33,525kN \quad \left| \begin{array}{l} x' = 0 \quad N_{\alpha_4} = -33,525kN \\ x' = 2m \quad N_{\alpha_4} = -6,705kN \end{array} \right.
 \end{aligned}$$

$$\begin{aligned}
 T_{\alpha_4} &= -R_B \cos \alpha + q \cdot x' \cos \alpha = -37,5kN \cos \alpha + 15 \frac{kN}{m} x' \cos \alpha = \\
 &= 6,705 \frac{kN}{m} x' - 16,763kN \quad \left| \begin{array}{l} x' = 0 \quad N_{\alpha_4} = -16,763kN \\ x' = 2m \quad N_{\alpha_4} = -3,353kN \end{array} \right.
 \end{aligned}$$

$$\begin{aligned}
 M_{\alpha_4} &= R_B \cdot x' - 15kN/m \cdot x' \cdot \frac{x'}{2} = \\
 &= -7,5 \frac{kN}{m} x'^2 + 37,5kN \cdot x' \quad \left| \begin{array}{l} x' = 0 \quad M_{\alpha_4} = 0 \\ x' = 2m \quad M_{\alpha_4} = 45kNm \end{array} \right.
 \end{aligned}$$

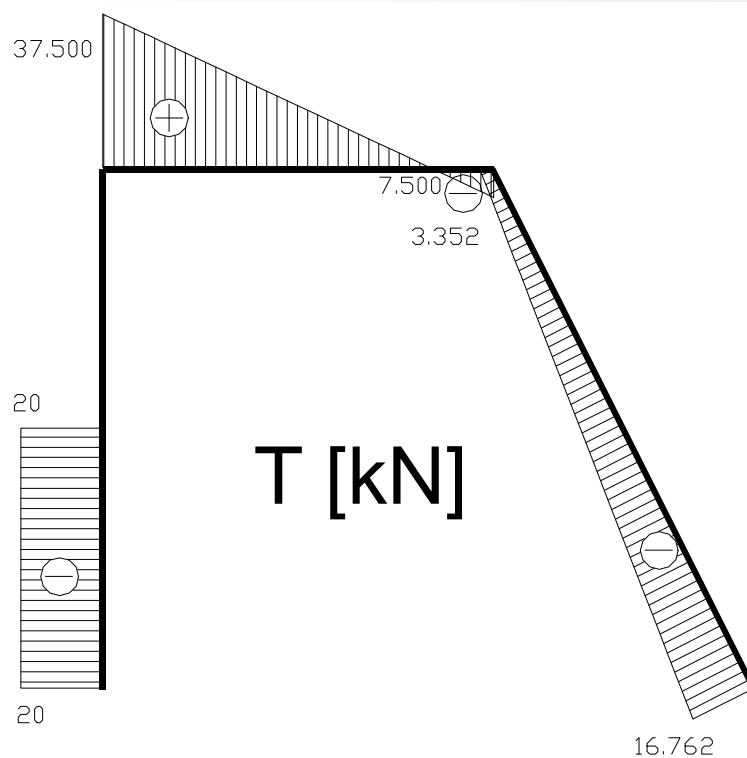
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Wykres sił normalnych



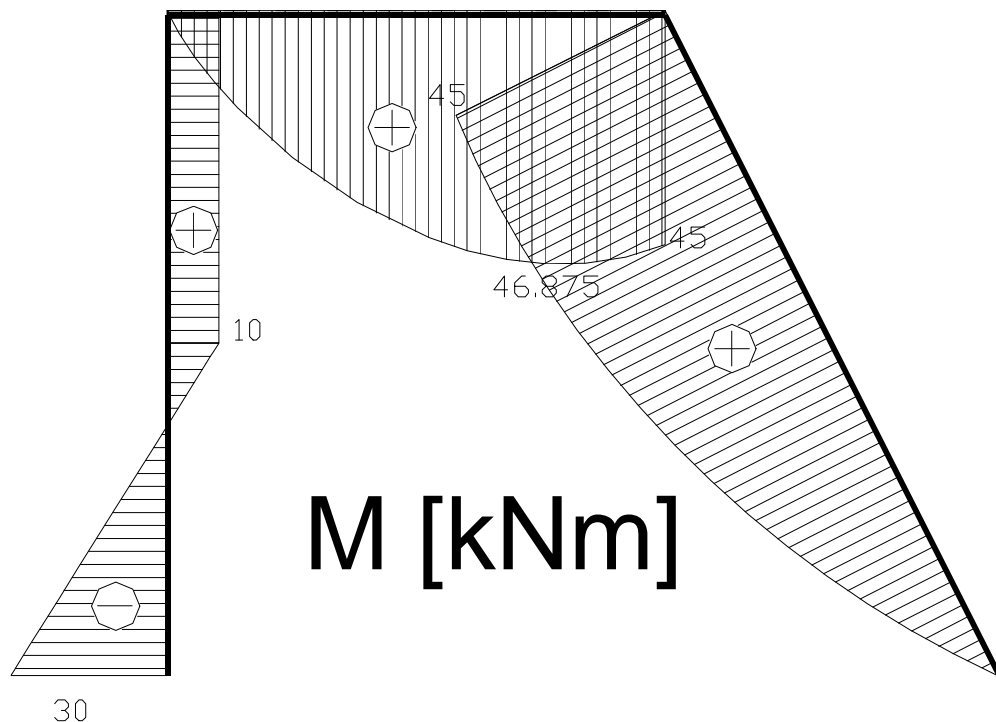
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Wykres sił tnących



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Wykres momentów zginających



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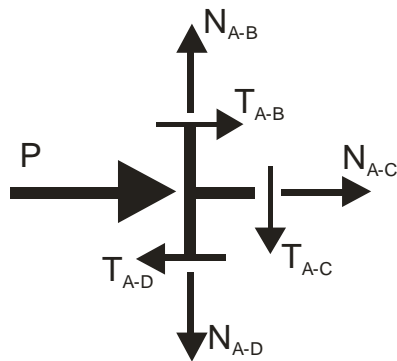
Kontrola wyników

- Sprawdzenie równowagi ogólnej
 - Suma rzutów sił na nową oś, nierównoległą do wcześniej wykorzystywanych.
 - Suma momentów względem dowolnego punktu na płaszczyźnie. Punkt powinien być dobrany tak, aby jak najwięcej reakcji znalazło się w równaniu.

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Kontrola wyników

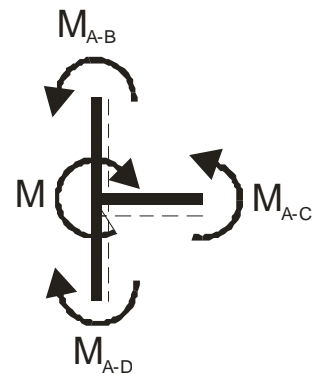
■ Sprawdzenie równowagi w węzłach:



$$\sum X : T_{A-B} + N_{A-C} - T_{A-D} + P = 0$$

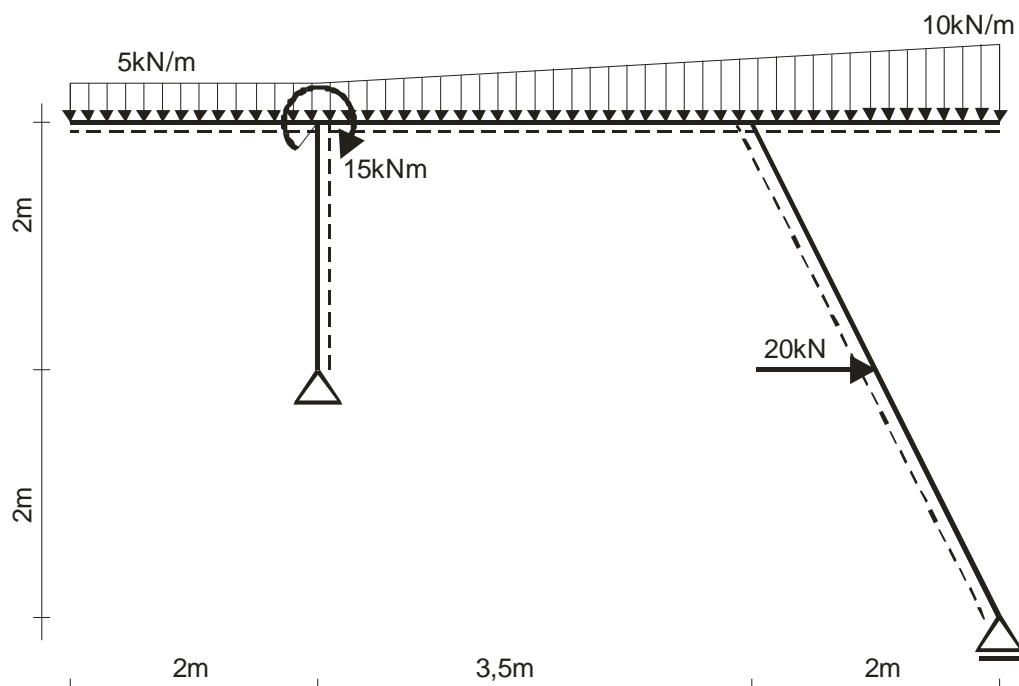
$$\sum Y : N_{A-B} - T_{A-C} - N_{A-D} = 0$$

$$\sum M_A : M_{A-B} + M_{A-C} - M_{A-D} - M = 0$$



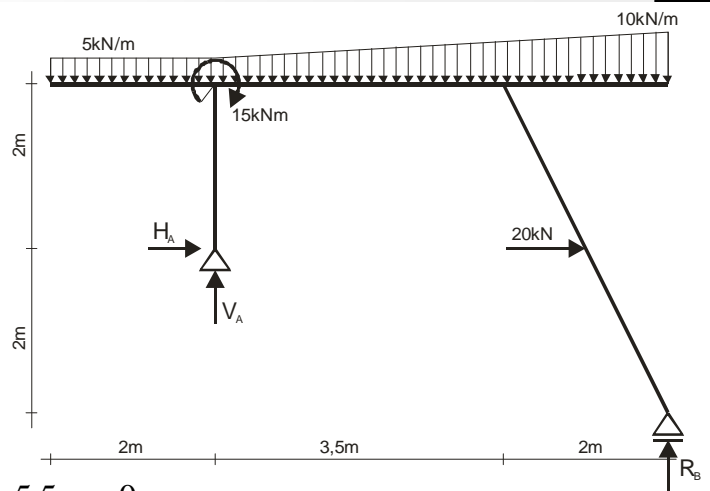
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Przykład – rama



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Reakcje



$$\sum X : H_A + 20kN = 0$$

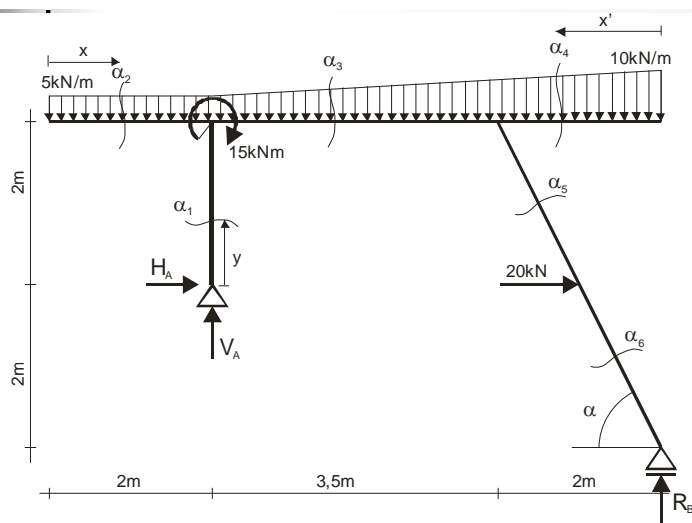
$$\sum Y : V_A + R_B - 5 \frac{kN}{m} \cdot 7,5m - \frac{1}{2} 5 \frac{kN}{m} \cdot 5,5m = 0$$

$$\sum M_A : R_B \cdot 5,5m - 15kNm + 5 \frac{kN}{m} \cdot 2m \cdot \frac{1}{2} 2m - 5 \frac{kN}{m} \cdot 5,5m \cdot \frac{1}{2} 5,5m +$$

$$- \frac{1}{2} 5 \frac{kN}{m} \cdot 5,5m \cdot \frac{2}{3} 5,5m = 0$$

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Przyjęcie przekrojów, przedziały



$$\alpha_1 - \alpha_1 \quad y \in \langle 0; 2m \rangle$$

$$\alpha_2 - \alpha_2 \quad x \in \langle 0; 2m \rangle$$

$$\alpha_3 - \alpha_3 \quad x \in \langle 2m; 5,5m \rangle$$

$$\alpha_4 - \alpha_4 \quad x \in \langle 5,5m; 7,5m \rangle$$

$$x' \in \langle 0; 2m \rangle$$

$$\alpha_5 - \alpha_5 \quad x \in \langle 5,5m; 6,5m \rangle \quad y \in \langle 0; 2m \rangle$$

$$x' \in \langle 1m; 2m \rangle$$

$$\alpha_6 - \alpha_6 \quad x \in \langle 6,5m; 7,5m \rangle \quad y \in \langle -2m; 0 \rangle$$

$$x' \in \langle 0; 1m \rangle$$

$$H_A = -20kN$$

$$V_A = 27,424kN$$

$$R_B = 23,826kN$$

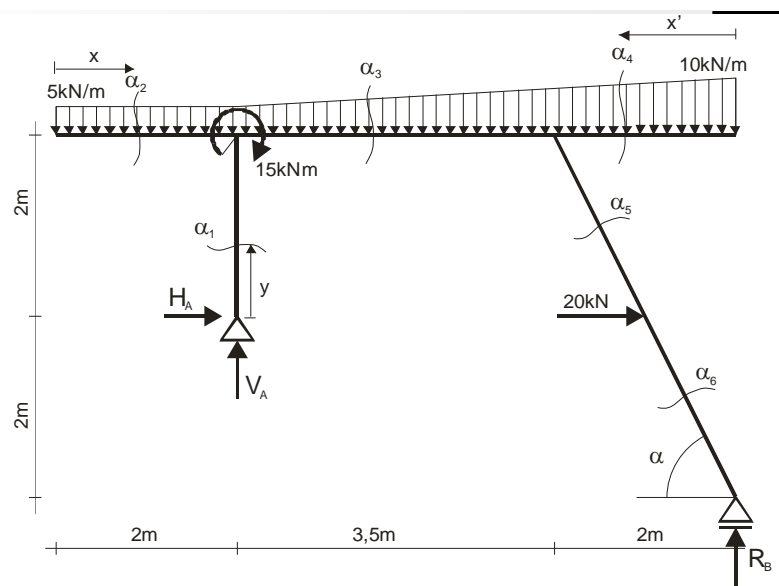
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Przekrój $\alpha_1 - \alpha_1$ $y \in \langle 0; 2m \rangle$

$$N_{\alpha_1} = -V_A = -27,424kN$$

$$T_{\alpha_1} = -H_A = 20kN$$

$$M_{\alpha_1} = -H_A \cdot y = 20kN \cdot y$$



$$\left| \begin{array}{l} y = 0 \quad M_{\alpha_1} = 0 \\ y = 2m \quad M_{\alpha_1} = 40kNm \end{array} \right.$$

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Przekrój $\alpha_2 - \alpha_2$ $x \in \langle 0; 2m \rangle$

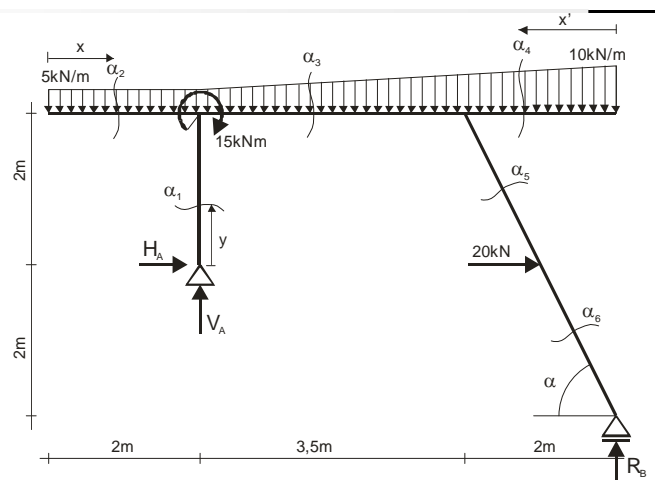
$$N_{\alpha_2} = 0$$

$$T_{\alpha_2} = -5 \frac{kN}{m} \cdot x$$

$$\left| \begin{array}{l} x = 0 \quad T_{\alpha_2} = 0 \\ x = 2m \quad T_{\alpha_2} = -10kN \end{array} \right.$$

$$M_{\alpha_2} = -5 \frac{kN}{m} \cdot x \cdot \frac{x}{2} = -2,5 \frac{kN}{m} \cdot x^2$$

$$\left| \begin{array}{l} x = 0 \quad M_{\alpha_2} = 0 \\ x = 2m \quad M_{\alpha_2} = -10kNm \end{array} \right.$$



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Przekrój $\alpha_3 - \alpha_3$ $x \in \langle 2m; 5,5m \rangle$

$$N_{\alpha_3} = -H_A = 20kN$$

$$\frac{5 \frac{kN}{m}}{q'} = \frac{5,5m}{x-2m} \quad q' = 0,909 \frac{kN}{m^2} (x-2m) = 0,909 \frac{kN}{m^2} x - 1,818 \frac{kN}{m}$$

$$\begin{aligned} T_{\alpha_3} &= V_A - 5 \frac{kN}{m} \cdot x - \frac{1}{2} q' (x-2m) = \\ &= 27,424kN - 5 \frac{kN}{m} \cdot x - \frac{1}{2} \left(0,909 \frac{kN}{m^2} x - 1,818 \frac{kN}{m} \right) (x-2m) = \\ &= -0,455 \frac{kN}{m^2} x^2 - 3,182 \frac{kN}{m} x + 25,606kN \end{aligned}$$

$$\left. \begin{array}{l} x = 2m \quad T_{\alpha_3} = 17,422kN \\ x = 5,5m \quad T_{\alpha_3} = -5,659kN \end{array} \right\}$$

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Przekrój $\alpha_3 - \alpha_3$ $x \in \langle 2m; 5,5m \rangle$

$$\begin{aligned} M_{\alpha_3} &= V_A \cdot (x-2m) - H_A \cdot 2m - 5 \frac{kN}{m} \cdot x \cdot \frac{x}{2} + \\ &\quad - \frac{1}{2} q' \cdot (x-2m) \cdot \frac{1}{3} (x-2m) + 15kNm = \\ &= 27,424kN \cdot (x-2m) + 20kN \cdot 2m - 2,5 \frac{kN}{m} \cdot x^2 + \\ &\quad - \frac{1}{6} \left(0,909 \frac{kN}{m^2} x - 1,818 \frac{kN}{m} \right) \cdot (x-2m)^2 + 15kNm = \\ &= -0,156 \frac{kN}{m^2} x^3 - 1,591 \frac{kN}{m} x^2 + 25,606kN \cdot x + 1,364kNm \end{aligned}$$

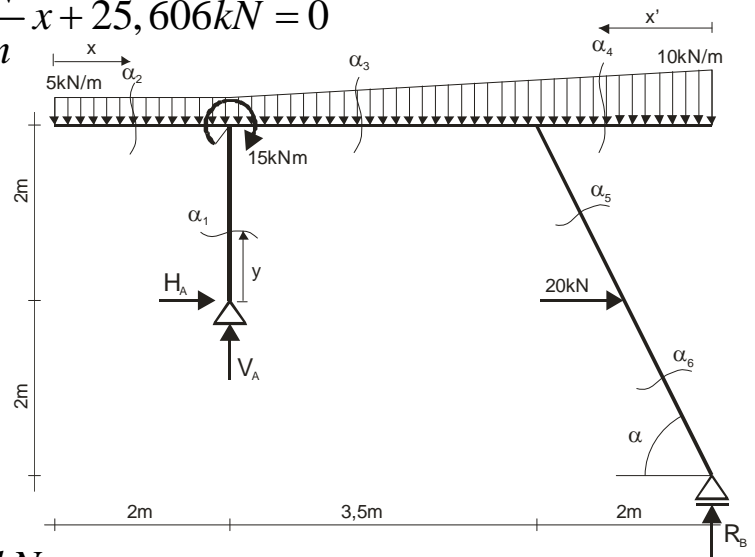
$$\left. \begin{array}{l} x = 2m \quad M_{\alpha_3} = 44,964kNm \\ x = 5,5m \quad M_{\alpha_3} = 68,115kNm \end{array} \right\}$$

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Ekstremum

$$T_{\alpha_3} = -0,455 \frac{kN}{m^2} x^2 - 3,182 \frac{kN}{m} x + 25,606 kN = 0$$

$$x = 4,78m$$



$$M_{\alpha_3} = -0,156 \frac{kN}{m^2} x^3 - 1,591 \frac{kN}{m} x^2 + 25,606 kN \cdot x + 1,364 kNm$$

$$M_{\alpha_3}(4,78m) = 70,371 kNm$$

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Przekrój $\alpha_4 - \alpha_4$ $x \in \langle 5,5m; 7,5m \rangle$

$$N_{\alpha_4} = 0$$

$$T_{\alpha_4} = 5 \frac{kN}{m} \cdot (7,5m - x) + q'(7,5m - x) + \frac{1}{2} \left(5 \frac{kN}{m} - q' \right) (7,5m - x) =$$

$$= 37,5kN - 5 \frac{kN}{m} \cdot x - \left(0,909 \frac{kN}{m^2} x - 1,818 \frac{kN}{m} \right) (7,5m - x) +$$

$$+ \frac{1}{2} \left(5 \frac{kN}{m} - 0,909 \frac{kN}{m^2} x + 1,808 \frac{kN}{m} \right) (7,5m - x) =$$

$$= -0,454 \frac{kN}{m^2} x^2 - 3,182 \frac{kN}{m} x + 49,433 kN$$

$$\left| \begin{array}{l} x = 5,5m \quad T_{\alpha_4} = 18,199 kN \\ x = 7,5m \quad T_{\alpha_4} = 0,031 kN \approx 0 \end{array} \right.$$

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Przekrój $\alpha_4 - \alpha_4$ $x \in \langle 5,5m; 7,5m \rangle$

$$\begin{aligned}
 M_{\alpha_4} &= -5 \frac{kN}{m} \cdot (7,5m - x) \frac{1}{2} (7,5m - x) - q' (7,5m - x) \frac{1}{2} (7,5m - x) + \\
 &\quad - \frac{1}{2} \left(5 \frac{kN}{m} - q' \right) (7,5m - x) \frac{2}{3} (7,5m - x) = \\
 &= -5 \frac{kN}{m} \cdot \frac{1}{2} (7,5m - x)^2 - \left(0,909 \frac{kN}{m^2} - 1,818 \frac{kN}{m} \right) \frac{1}{2} (7,5m - x)^2 + \\
 &\quad - \frac{1}{2} \left(5 \frac{kN}{m} - \left(0,909 \frac{kN}{m^2} - 1,818 \frac{kN}{m} \right) \right) \frac{2}{3} (7,5m - x)^2 = \\
 &= -0,152 \frac{kN}{m^2} x^3 - 1,591 \frac{kN}{m} x^2 + 49,433 kN \cdot x - 217,331 kNm
 \end{aligned}$$

$$\left| \begin{array}{ll}
 x = 5,5m & M_{\alpha_4} = -18,886 kNm \\
 x = 7,5m & M_{\alpha_4} = -0,202 kNm \approx 0
 \end{array} \right.$$

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Przekrój $\alpha_4 - \alpha_4$

- Zwiększenie dokładności przy x w najwyższej potędze (większa liczba cyfr znaczących):

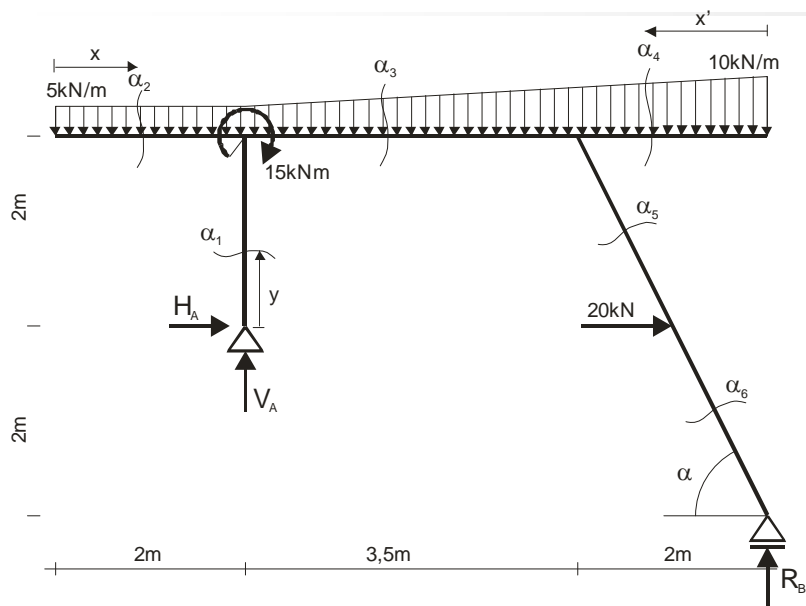
$$T_{\alpha_4} = -0,4545 \frac{kN}{m^2} x^2 - 3,182 \frac{kN}{m} x + 49,433 kN$$

$$M_{\alpha_4} = -0,1515 \frac{kN}{m^2} x^3 - 1,591 \frac{kN}{m} x^2 + 49,433 kN \cdot x - 217,331 kNm$$

$$\left| \begin{array}{lll}
 x = 5,5m & T_{\alpha_4} = 18,183 kN & M_{\alpha_4} = -18,783 kNm \\
 x = 7,5m & T_{\alpha_4} = 0,002 kN \approx 0 & M_{\alpha_4} = 0,009 kNm
 \end{array} \right.$$

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Przekrój $\alpha_6 - \alpha_6$ $x' \in \langle 0; 1m \rangle$



$$\sin \alpha = \frac{4m}{\sqrt{(4m)^2 + (2m)^2}} = 0,894 \quad \cos \alpha = \frac{2m}{\sqrt{(4m)^2 + (2m)^2}} = 0,447$$

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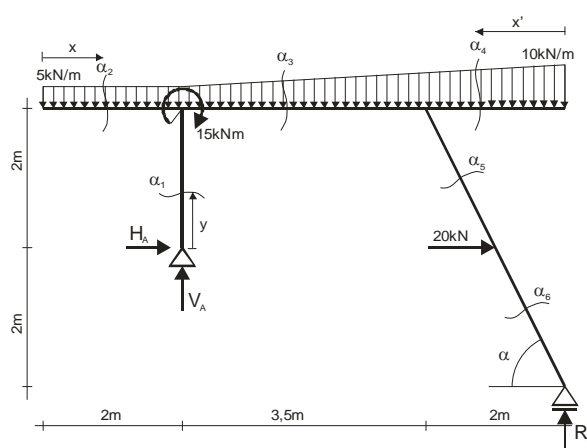
Przekrój $\alpha_6 - \alpha_6$ $x' \in \langle 0; 1m \rangle$

$$N_{\alpha_6} = -R_B \cdot \sin \alpha = -23,826kN \cdot 0,894 = -21,300kN$$

$$T_{\alpha_6} = -R_B \cdot \cos \alpha = -23,826kN \cdot 0,447 = -10,650kN$$

$$M_{\alpha_6} = R_B \cdot x' = 23,826kN \cdot x'$$

$$\left| \begin{array}{ll} x' = 0 & M_{\alpha_6} = 0 \\ x' = 1m & M_{\alpha_6} = 23,826kNm \end{array} \right.$$



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Przekrój $\alpha_5 - \alpha_5$ $x' \in \langle 1m; 2m \rangle$

$$N_{\alpha_5} = -R_B \cdot \sin \alpha + 20 \cos \alpha =$$

$$= -23,826kN \cdot 0,894 + 20kN \cdot 0,447 = -12,360kN$$

$$T_{\alpha_5} = -R_B \cdot \cos \alpha - 20 \sin \alpha =$$

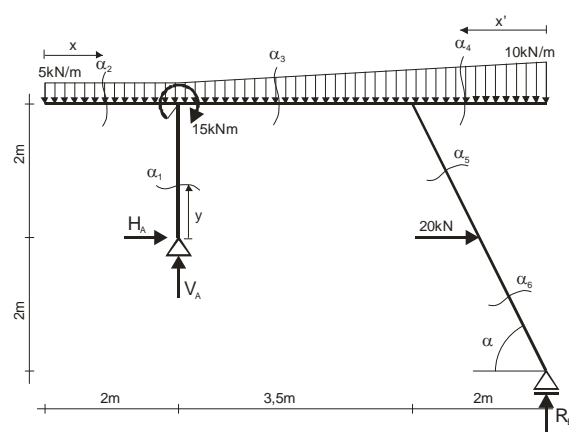
$$= -23,826kN \cdot 0,447 - 20kN \cdot 0,894 = -28,530kN$$

$$M_{\alpha_5} = R_B \cdot x' + 20 \cdot y =$$

$$= 23,826kN \cdot x' + 20kN \cdot y$$

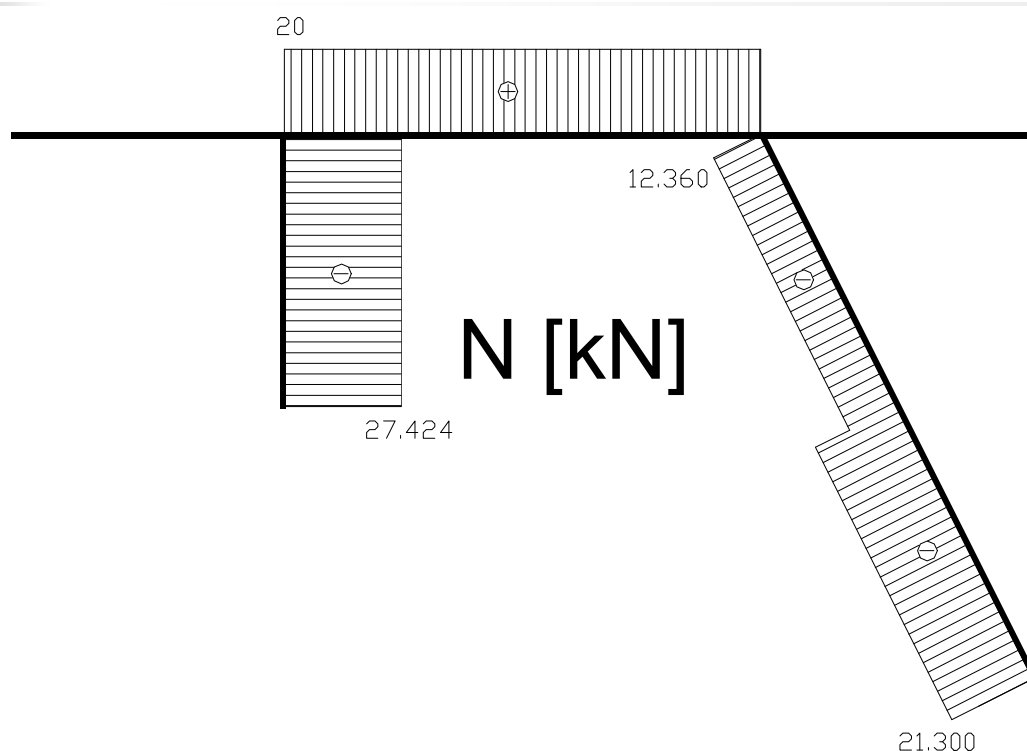
$$x' = 1m \quad y = 0 \quad M_{\alpha_5} = 23,826kNm$$

$$x' = 2m \quad y = 2m \quad M_{\alpha_5} = 87,652kNm$$



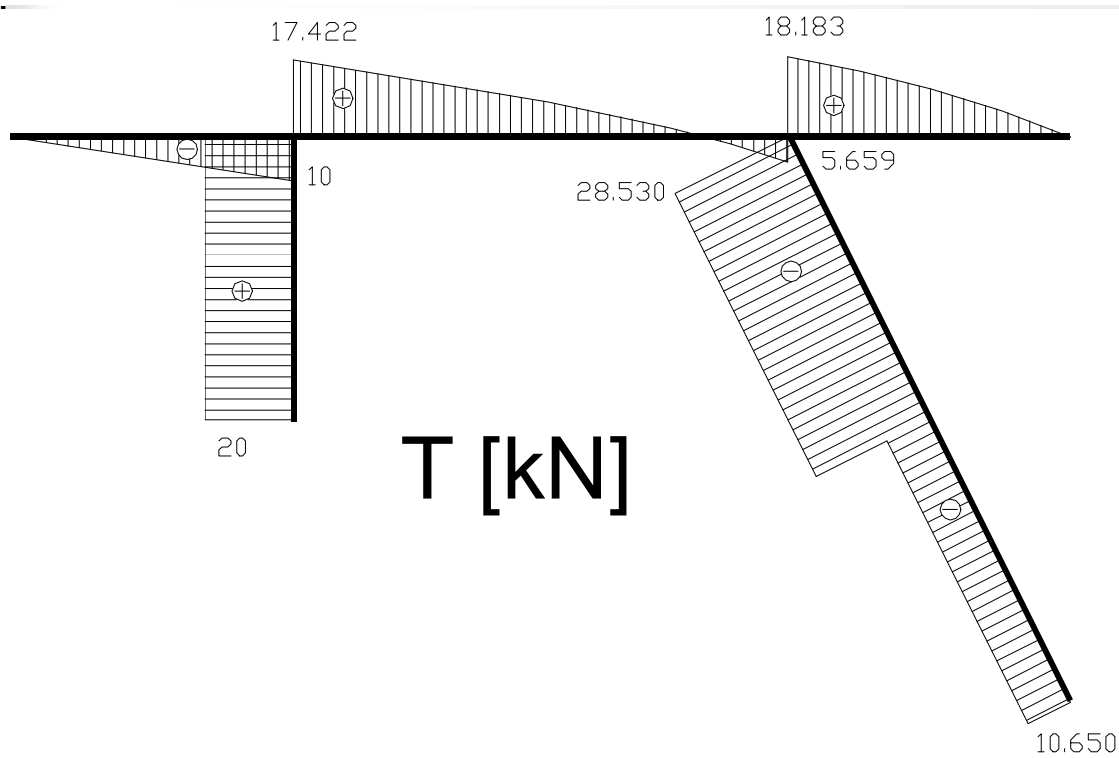
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Wykres sił normalnych

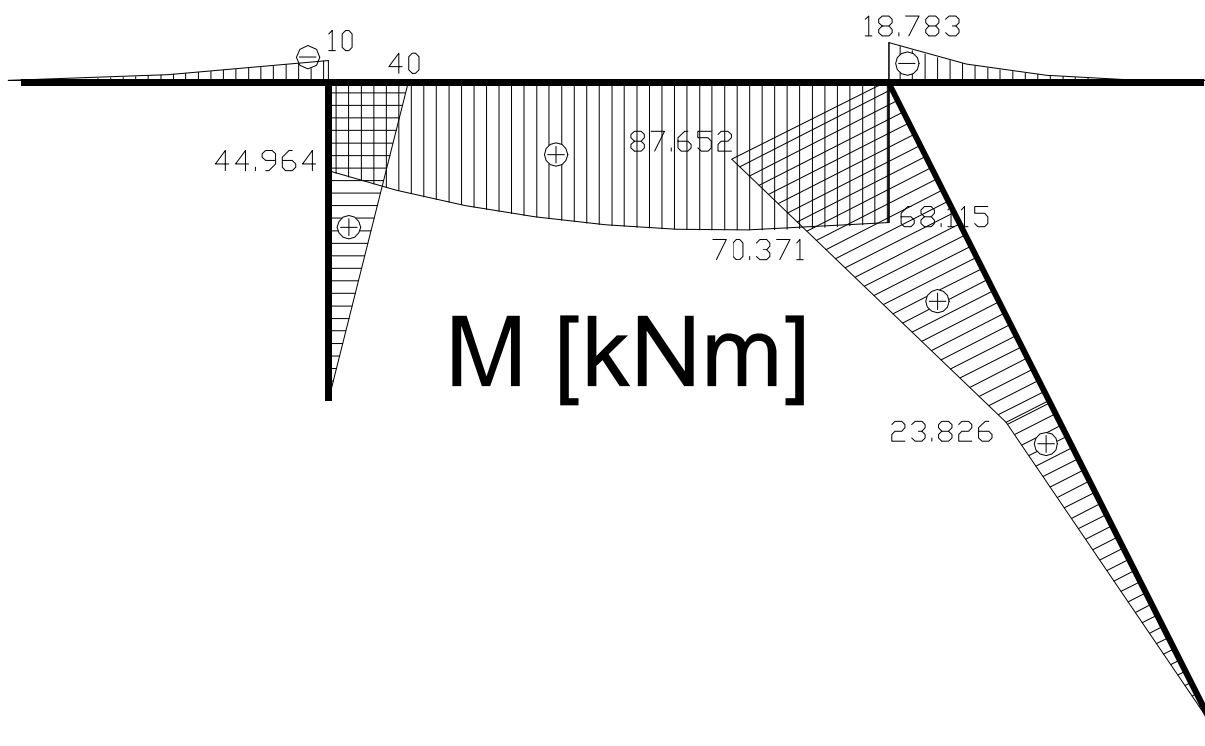


32

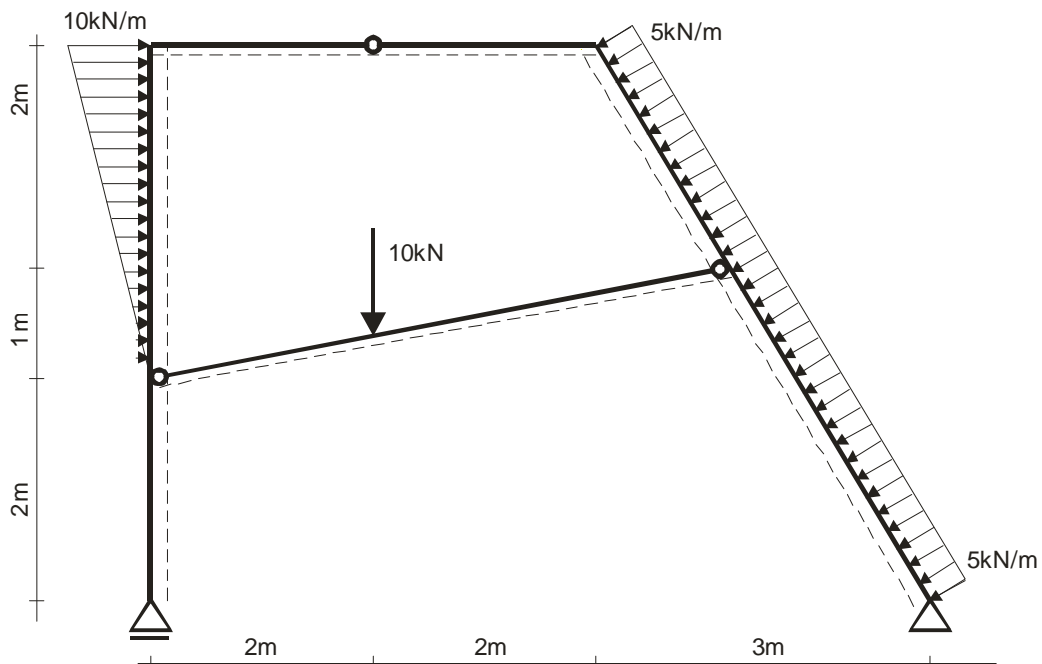
Wykres sił tnących



Wykres momentów zginających

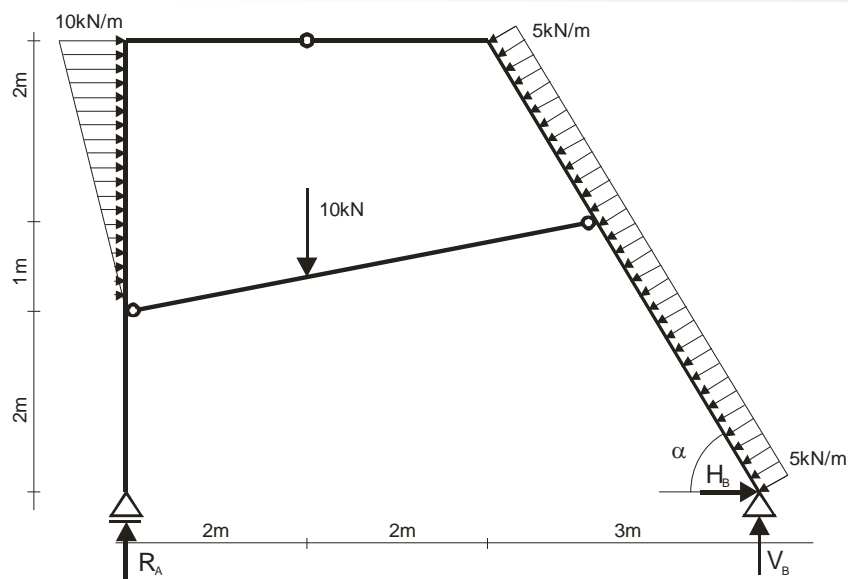


Przykład – rama ze ściągiem



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Reakcje podporowe



$$\sin \alpha = \frac{5m}{\sqrt{(3m)^2 + (5m)^2}} = 0,857 \quad \cos \alpha = \frac{3m}{\sqrt{(3m)^2 + (5m)^2}} = 0,514$$

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Reakcje podporowe

$$\sum X : H_B + \frac{1}{2} 10 \frac{kN}{m} \cdot 3m - 5 \frac{kN}{m} \cdot \sqrt{(3m)^2 + (5m)^2} \sin \alpha = 0$$

$$\sum Y : R_A + V_B - 10kN - 5 \frac{kN}{m} \cdot \sqrt{(3m)^2 + (5m)^2} \cos \alpha = 0$$

$$\sum M_B : R_A \cdot 7m + \frac{1}{2} 10 \frac{kN}{m} \cdot 3m \cdot \left(2m + \frac{2}{3} 3m \right) - 10kN \cdot 5m +$$

$$- 5 \frac{kN}{m} \cdot \sqrt{(3m)^2 + (5m)^2} \cdot \frac{1}{2} \sqrt{(3m)^2 + (5m)^2} = 0$$

$$H_B = 9,986kN$$

$$R_A = 10,714kN$$

$$V_B = 14,272kN$$

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Reakcje w przegubach ściągu

$$\sum X : H_D - H_E = 0 \quad \sum Y : V_D + V_E - 10kN = 0$$

$$H_D = 2,247kN$$

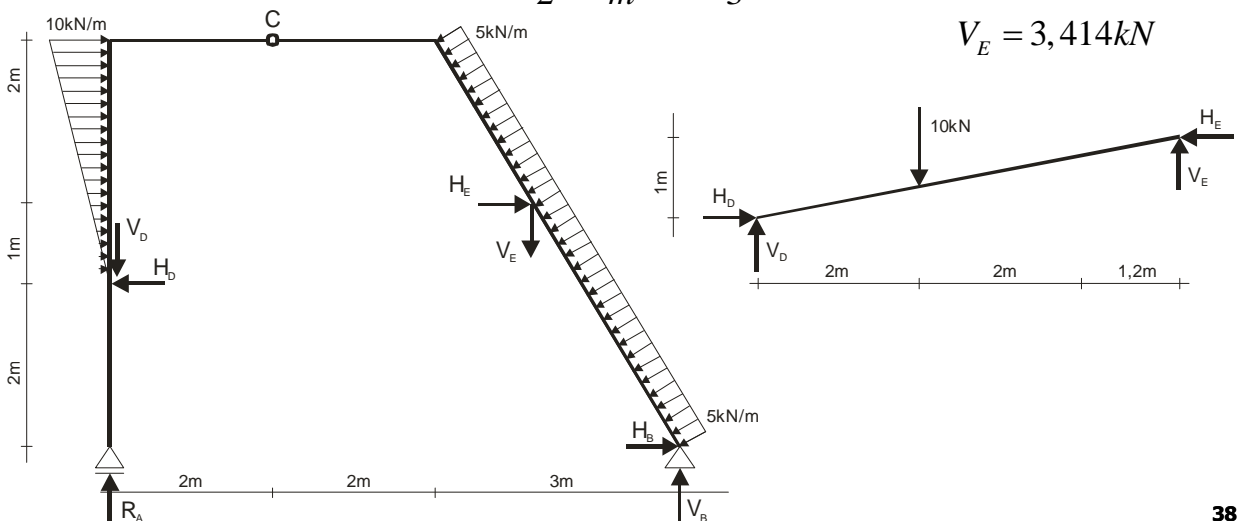
$$\sum M_D : V_E \cdot 5,2m + H_E \cdot 1m - 10kN \cdot 2m = 0$$

$$H_E = 2,247kN$$

$$\sum M_C : R_A \cdot 2m - V_D \cdot 2m + H_D \cdot 3m - \frac{1}{2} 10 \frac{kN}{m} \cdot 3m \cdot \frac{1}{3} 3m = 0$$

$$V_D = 6,586kN$$

$$V_E = 3,414kN$$



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Przyjęcie przekrojów, przedziały

$$\alpha_1 - \alpha_1 \quad y \in \langle 0; 2m \rangle$$

$$\alpha_2 - \alpha_2 \quad y \in \langle 2m; 5m \rangle$$

$$\alpha_3 - \alpha_3 \quad x \in \langle 0; 4m \rangle$$

$$\alpha_4 - \alpha_4 \quad x \in \langle 4m; 5,2m \rangle \quad y \in \langle 3m; 5m \rangle$$

$$x' \in \langle 1,8m; 3m \rangle$$

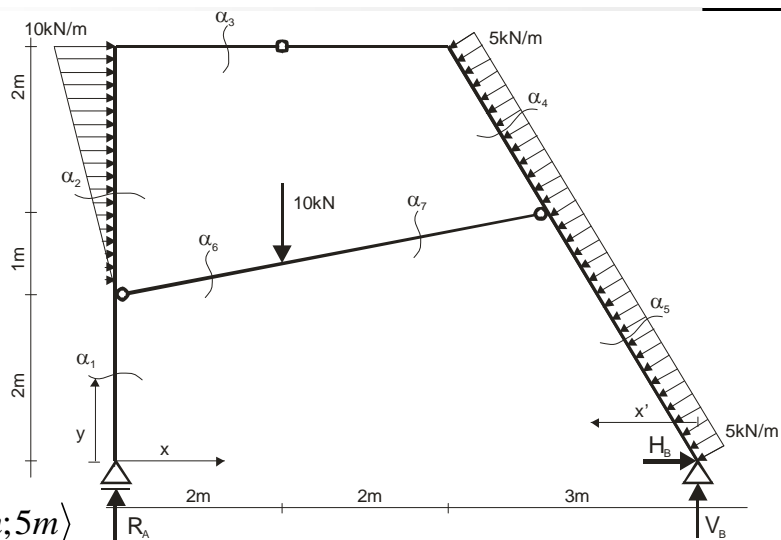
$$\alpha_5 - \alpha_5 \quad x \in \langle 5,2m; 7m \rangle \quad y \in \langle 0; 3m \rangle$$

$$x' \in \langle 0; 1,8m \rangle$$

$$\alpha_6 - \alpha_6 \quad x \in \langle 0; 2m \rangle \quad y \in \langle 2m; 2,6m \rangle$$

$$\alpha_7 - \alpha_7 \quad x \in \langle 2m; 5,2m \rangle \quad y \in \langle 2,6m; 3m \rangle$$

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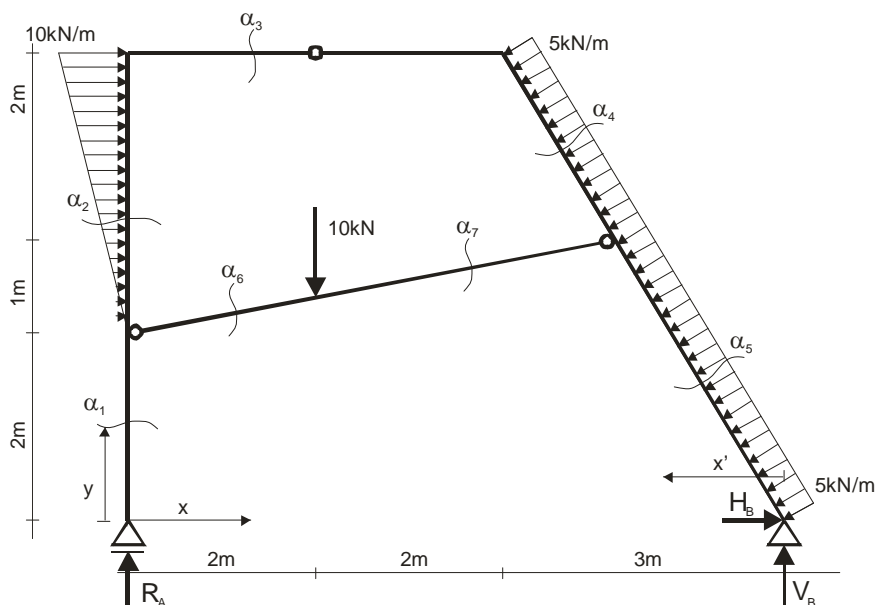


Przekrój $\alpha_1 - \alpha_1 \quad y \in \langle 0; 2m \rangle$

$$N_{\alpha_1} = -R_A = -10,714kN$$

$$T_{\alpha_1} = 0$$

$$M_{\alpha_1} = 0$$



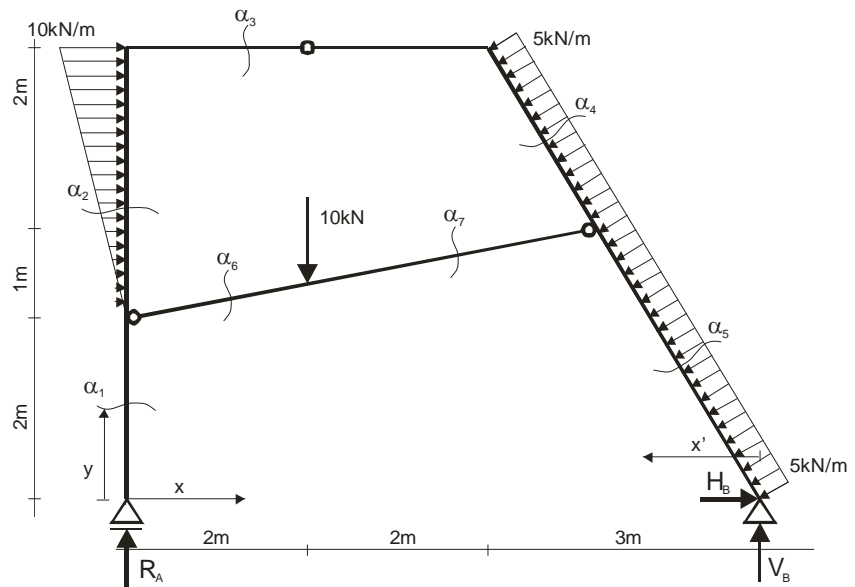
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Przekrój $\alpha_2 - \alpha_2$ $y \in \langle 2m; 5m \rangle$

$$N_{\alpha_2} = -R_A + V_D = -10,714kN + 6,586kN = -4,128kN$$

$$\frac{10 \frac{kN}{m}}{q'} = \frac{3m}{y-2m}$$

$$q' = 3,333 \frac{kN}{m^2} (y-2m)$$



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Przekrój $\alpha_2 - \alpha_2$ $y \in \langle 2m; 5m \rangle$

$$T_{\alpha_2} = H_D - \frac{1}{2} q' \cdot (y-2m)$$

$$= 2,247kN - \frac{1}{2} 3,333 \frac{kN}{m^2} \cdot (y-2m)^2 =$$

$$= -1,667 \frac{kN}{m^2} y^2 + 6,666kN \cdot y - 4,419kN$$

$$\left| \begin{array}{l} y = 2m \quad T_{\alpha_2} = 2,245kN \\ y = 5m \quad T_{\alpha_2} = -12,764kN \end{array} \right.$$

$$M_{\alpha_2} = H_D \cdot (y-2m) - \frac{1}{2} q' \cdot (y-2m) \cdot \frac{1}{3} (y-2m) =$$

$$= 2,247kN \cdot (y-2m) - \frac{1}{6} 3,333 \frac{kN}{m^2} (y-2m)^3 =$$

$$= -0,5555 \frac{kN}{m^2} \cdot y^3 + 3,333 \frac{kN}{m} \cdot y^2 - 4,419kN \cdot y - 0,050kNm$$

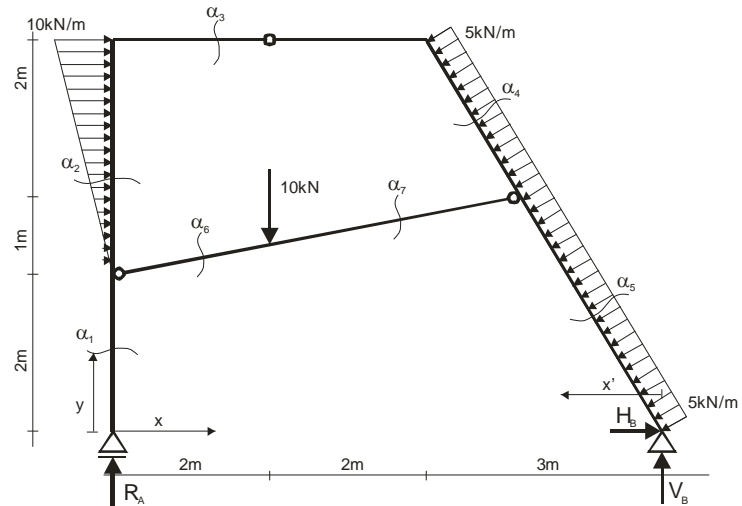
$$\left| \begin{array}{l} y = 2m \quad M_{\alpha_2} = 0 \\ y = 5m \quad M_{\alpha_2} = -8,258kNm \end{array} \right.$$

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Ekstremum

$$T_{\alpha_2} = -1,667 \frac{kN}{m^2} y^2 + 6,666 kN \cdot y - 4,419 kN = 0$$

$$y = 3.16m$$



$$M_{\alpha_2} = -0,5555 \frac{kN}{m^2} \cdot y^3 + 3,333 \frac{kN}{m} \cdot y^2 - 4,419 kN \cdot y - 0,050 kNm$$

$$M_{\alpha_2}(3,16m) = 1.739 kNm$$

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Przekrój $\alpha_3 - \alpha_3 \quad x \in \langle 0; 4m \rangle$

$$\begin{aligned} N_{\alpha_3} &= H_D - \frac{1}{2} 10 \frac{kN}{m} \cdot 3m = \\ &= 2,247 kN - 15 kN = -12,753 kN \end{aligned}$$

$$T_{\alpha_3} = R_A - V_D = 10,714 kN - 6,586 kN = 4,128 kN$$

$$\begin{aligned} M_{\alpha_3} &= R_A \cdot x - V_D \cdot x + H_D \cdot 3m - \frac{1}{2} 10 \frac{kN}{m} \cdot 3m \cdot \frac{1}{3} 3m = \\ &= 10,714 kN \cdot x - 6,586 kN \cdot x + 2,247 kN \cdot 3m - 15 kNm = \\ &= 4,128 kN \cdot x - 8,259 kNm \end{aligned}$$

$$x = 0m \quad M_{\alpha_3} = -8,259 kNm$$

$$x = 2m \quad M_{\alpha_3} = -0,003 kNm \approx 0$$

$$x = 4m \quad M_{\alpha_3} = 8,253 kNm$$

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Przekrój $\alpha_5 - \alpha_5$ $x' \in \langle 0m; 1,8m \rangle$

$$\begin{aligned} N_{\alpha_5} &= H_B \cos \alpha - V_B \sin \alpha = \\ &= 9,986kN \cdot 0,514 - 14,272kN \cdot 0,857 = -7,098kN \end{aligned}$$

$$\begin{aligned} T_{\alpha_5} &= -H_B \sin \alpha - V_B \cos \alpha + 5 \frac{kN}{m} \sqrt{x'^2 + y^2} = \\ &= -9,986kN \cdot 0,857 - 14,272kN \cdot 0,514 + 5 \frac{kN}{m} \sqrt{x'^2 + y^2} = \\ &= -15,894kN + 5 \frac{kN}{m} \sqrt{x'^2 + y^2} = \\ &= -15,894kN + 9,720 \frac{kN}{m} x' \quad \left| \begin{array}{l} x' = 0m \quad T_{\alpha_5} = -15,894kN \\ x' = 1,8m \quad T_{\alpha_5} = 1,602kN \end{array} \right. \end{aligned}$$

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Przekrój $\alpha_5 - \alpha_5$ $x' \in \langle 0m; 1,8m \rangle$

$$\begin{aligned} M_{\alpha_5} &= V_B \cdot x' + H_B \cdot y - 5 \frac{kN}{m} \sqrt{x'^2 + y^2} \cdot \frac{1}{2} \sqrt{x'^2 + y^2} = \\ &= 14,272kN \cdot x' + 9,986kN \cdot y - 2,5 \frac{kN}{m} (x'^2 + y^2) = \\ &= 14,272kN \cdot x' + 9,986kN \cdot 1,667x' - 2,5 \frac{kN}{m} (x'^2 + (1,667x')^2) = \\ &= -9,447 \frac{kN}{m} x'^2 + 30,919kN \cdot x' \end{aligned}$$

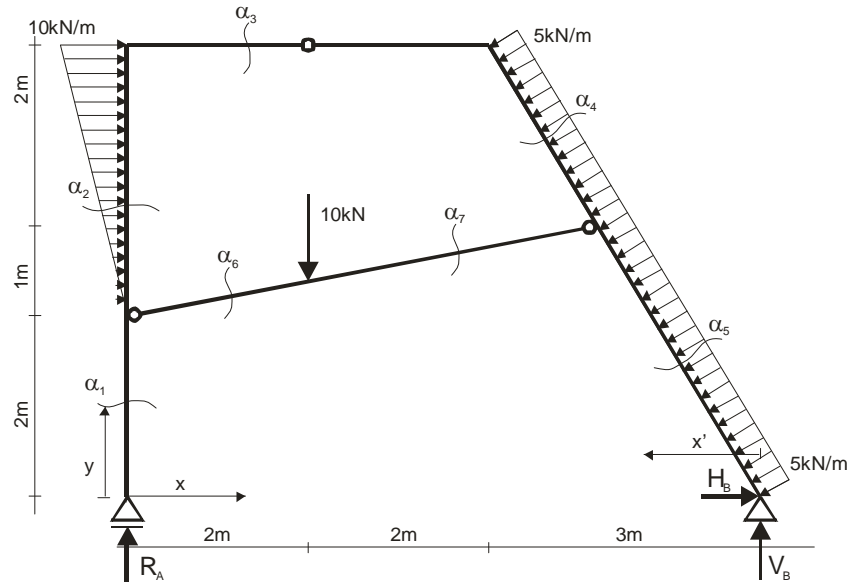
$$\left| \begin{array}{l} x' = 0 \quad M_{\alpha_5} = 0 \\ x' = 1,8m \quad M_{\alpha_5} = 25,046kNm \end{array} \right.$$

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Ekstremum

$$T_{\alpha_5} = -15,894kN + 9,720 \frac{kN}{m} x' = 0$$

$$x' = 1.635m$$



$$M_{\alpha_5} = -9,447 \frac{kN}{m} x'^2 + 30,919kN \cdot x'$$

$$M_{\alpha_5}(1.635m) = 25.299kNm$$

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Przekrój $\alpha_4 - \alpha_4$ $x' \in \langle 1,8m; 3m \rangle$

$$N_{\alpha_4} = -7,098kN + H_E \cos \alpha + V_E \sin \alpha =$$

$$= -7,098kN + 2,247kN \cdot 0,514 + 3,414kN \cdot 0,857 =$$

$$= -3,008kN$$

$$T_{\alpha_4} = -15,894kN + 9,720 \frac{kN}{m} x' - H_E \sin \alpha + V_E \cos \alpha =$$

$$= -15,894kN + 9,720 \frac{kN}{m} x' - 2,247kN \cdot 0,857 + 3,414kN \cdot 0,514 =$$

$$= 9,720 \frac{kN}{m} x' - 16,065kN$$

$$x' = 0 \quad T_{\alpha_4} = 1,431kN$$

$$x' = 1,8m \quad T_{\alpha_4} = 13,095kN$$

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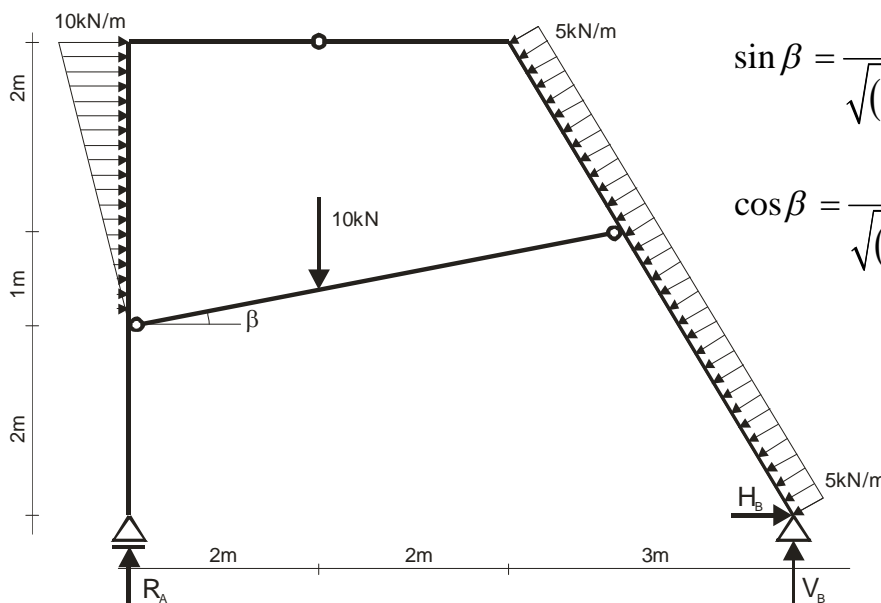
Przekrój $\alpha_4 - \alpha_4$ $x' \in \langle 1,8m; 3m \rangle$

$$\begin{aligned}
 M_{\alpha_4} &= -9,447 \frac{kN}{m} x'^2 + 30,919kN \cdot x' + H_E \cdot (y - 3m) - V_E \cdot (x' - 1,8m) = \\
 &= -9,447 \frac{kN}{m} x'^2 + 30,919kN \cdot x' + 2,247kN(1,667x' - 3m) + \\
 &\quad - 3,414kN \cdot (x' - 1,8m) = \\
 &= -9,447 \frac{kN}{m} x'^2 + 31,251kN \cdot x' - 0,596kNm
 \end{aligned}$$

$$\left| \begin{array}{l} x' = 1,8m \quad M_{\alpha_4} = 25,048kNm \\ x' = 3m \quad M_{\alpha_4} = 8,134kNm \end{array} \right.$$

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Przekrój $\alpha_6 - \alpha_6$ $x \in \langle 0; 2m \rangle$

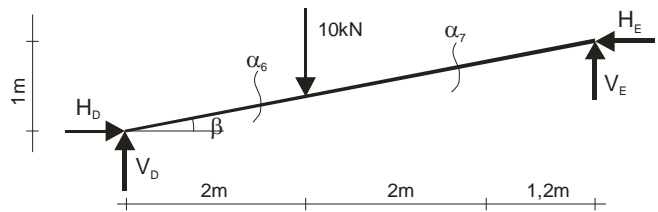


$$\sin \beta = \frac{1m}{\sqrt{(1m)^2 + (5,2m)^2}} = 0,189$$

$$\cos \beta = \frac{5,2m}{\sqrt{(1m)^2 + (5,2m)^2}} = 0,982$$

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Przekrój $\alpha_6 - \alpha_6$ $x \in \langle 0; 2m \rangle$



$$N_{\alpha_6} = -H_D \cos \beta - V_D \sin \beta =$$

$$= -2,247kN \cdot 0,982 - 6,586kN \cdot 0,189 = -3,451kN$$

$$T_{\alpha_6} = -H_D \sin \beta + V_D \cos \beta =$$

$$= -2,247kN \cdot 0,189 + 6,586kN \cdot 0,982 = 6,043kN$$

$$M_{\alpha_6} = -H_D \cdot (y - 2m) + V_D \cdot x =$$

$$= -2,247kN \cdot \left(\frac{x}{5,2} + 2m - 2m \right) + 6,586kN \cdot x =$$

$$= -0,432x + 6,586x = 6,154x$$

$$\left| \begin{array}{ll} x = 0m & M_{\alpha_6} = 0 \\ x = 2m & M_{\alpha_6} = 12,308kNm \end{array} \right.$$

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Przekrój $\alpha_7 - \alpha_7$ $x \in \langle 2m; 5,2m \rangle$

$$N_{\alpha_7} = -3,451kN + 10kN \sin \beta =$$

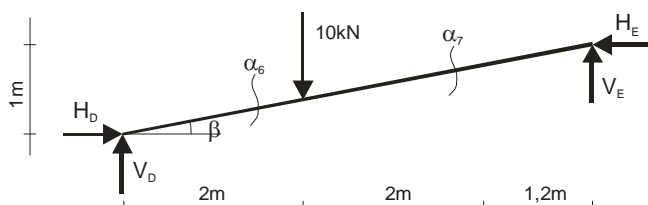
$$= -3,451kN + 10kN \cdot 0,189 = -1,561kN$$

$$T_{\alpha_7} = 6,043kN - 10kN \cos \beta =$$

$$= 6,043kN - 10kN \cdot 0,982 = -3,777kN$$

$$M_{\alpha_7} = 6,154x - 10kN \cdot (x - 2m) =$$

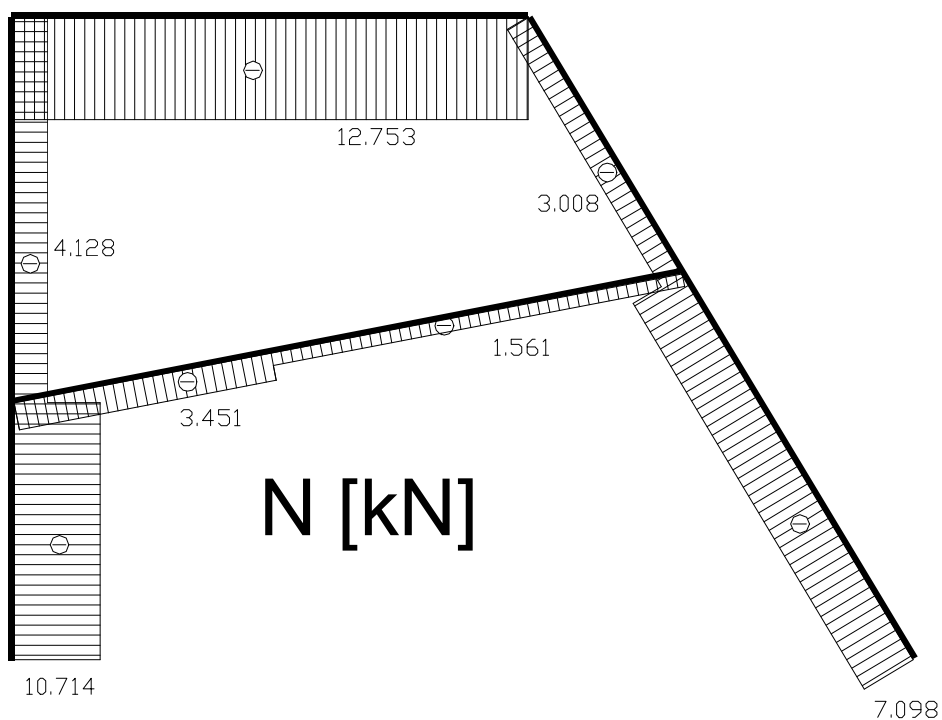
$$= -3,846x + 20kNm$$



$$\left| \begin{array}{ll} x = 2m & M_{\alpha_7} = 12,308kNm \\ x = 5,2m & M_{\alpha_7} = 0,001kNm \approx 0 \end{array} \right.$$

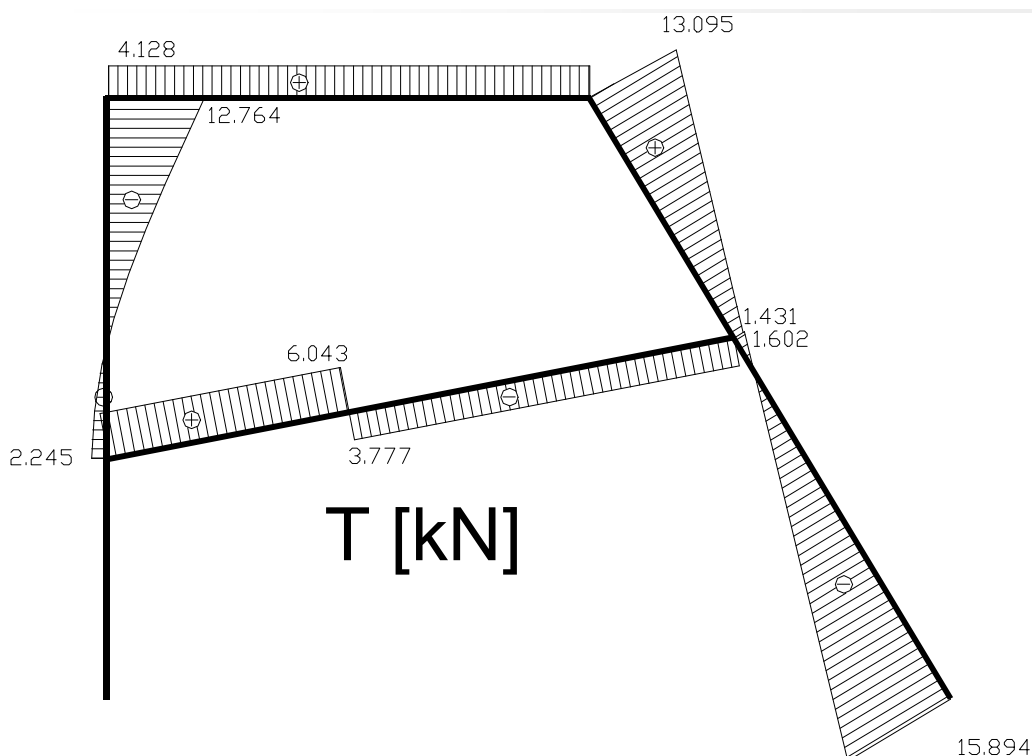
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Wykres sił normalnych



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Wykres sił tnących



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Wykres momentów zginających

