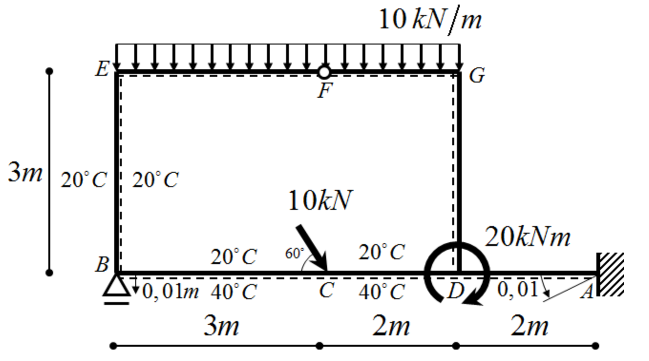
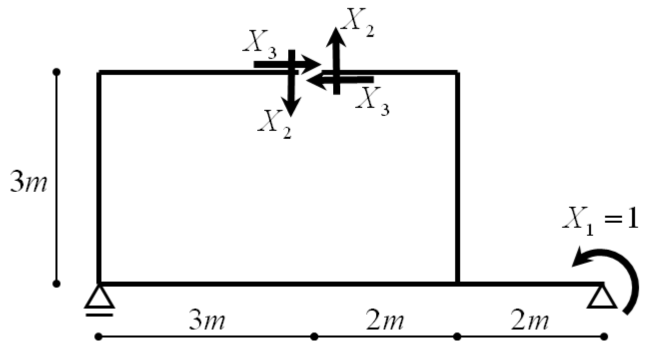


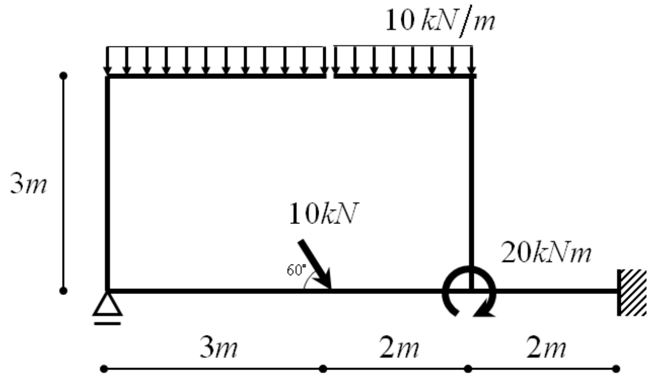
$E = 205GPa$ $\alpha_t = 1.2\cdot10^{-5}\cdot\frac{1}{K}$

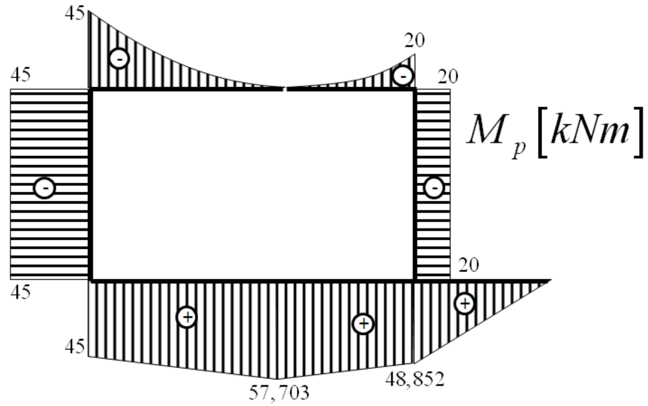
Przekrój: I200 $h = 0.2m$ $I_x = 2140cm^4$ $E\cdot I_x = 4387\,kN\cdot m^2$

Układ podstawowy metody sił (UPMS)

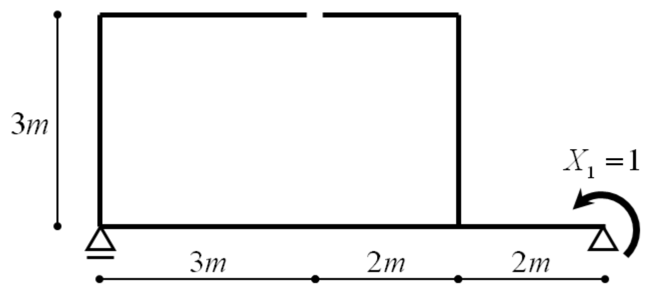


Stan "p"

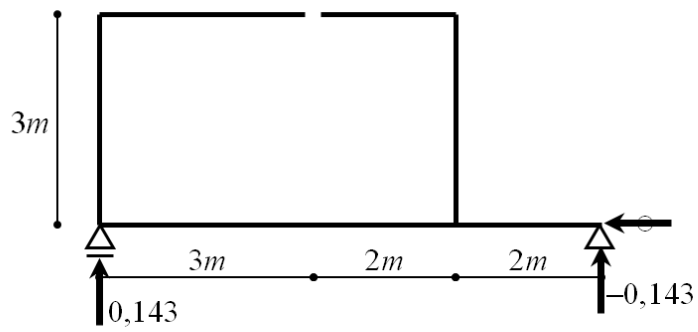


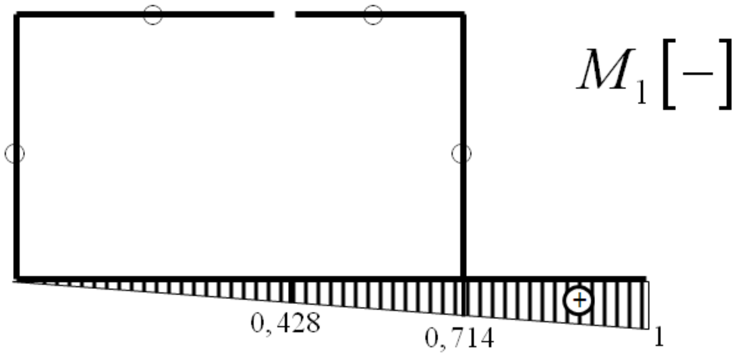


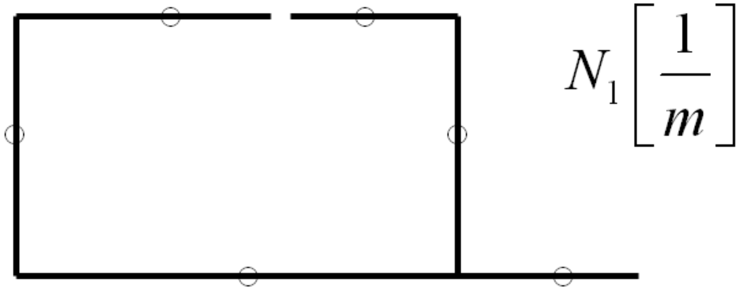
Stan X_1=1



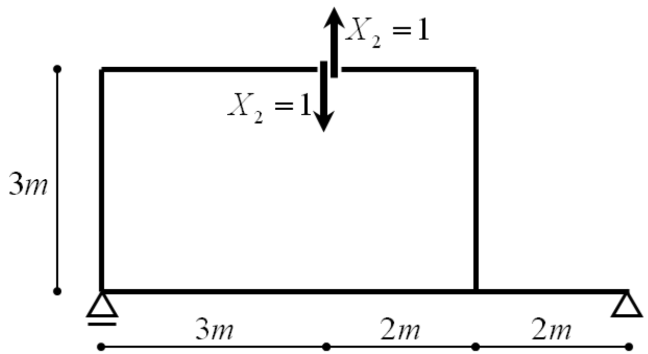
Reakcje w stanie X_1=1



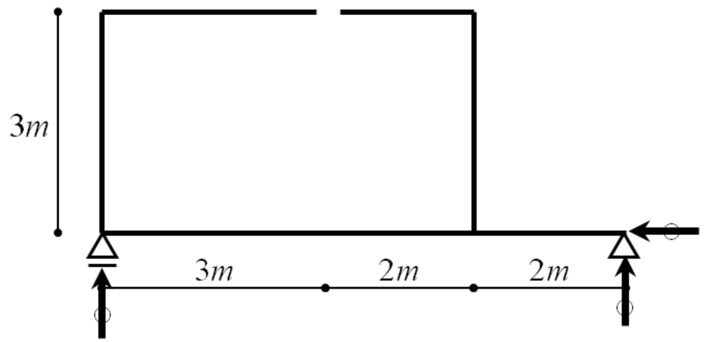


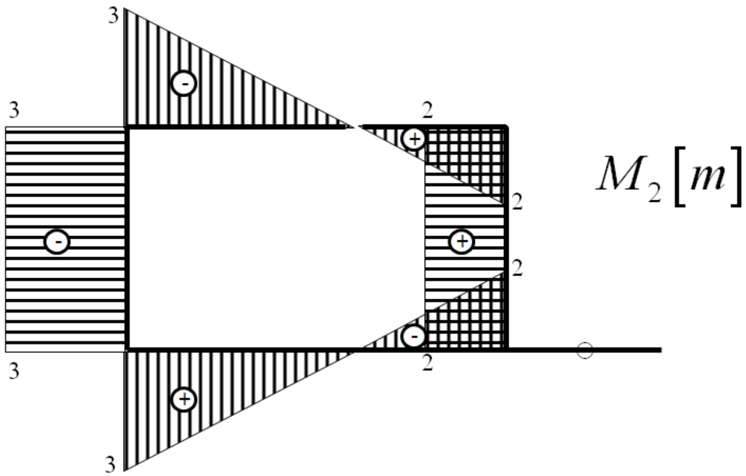


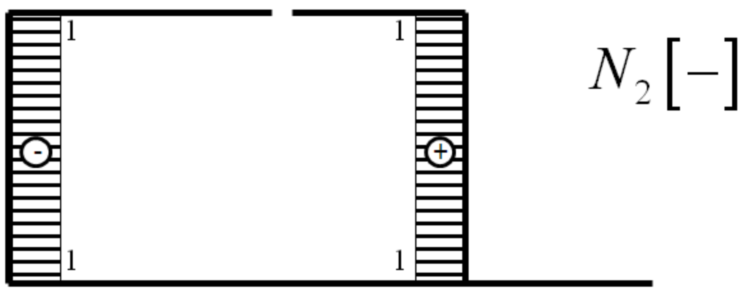
Stan X_2=1



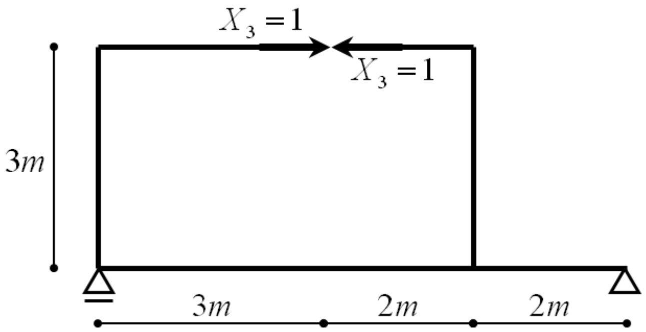
Reakcje w stanie X_2=1



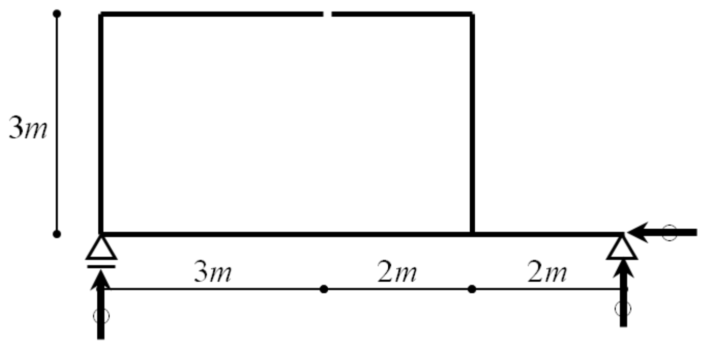


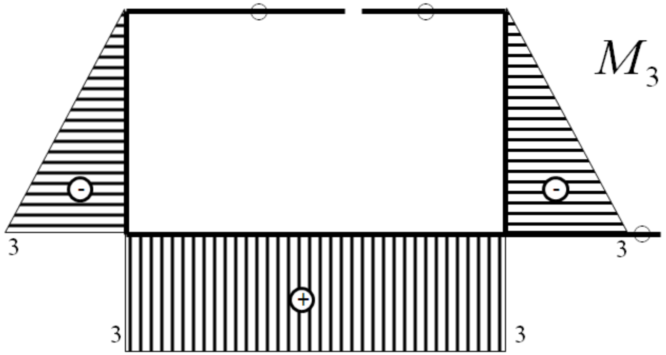


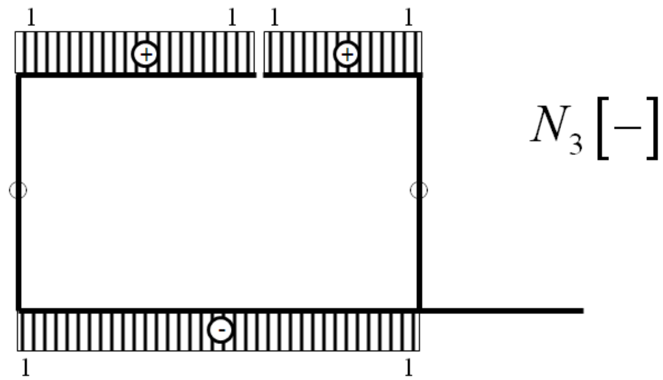
Stan X_3=1



Reakcje w stanie X_3=1

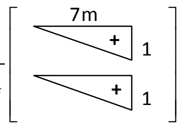






Wyznaczenie współczynników układu równań

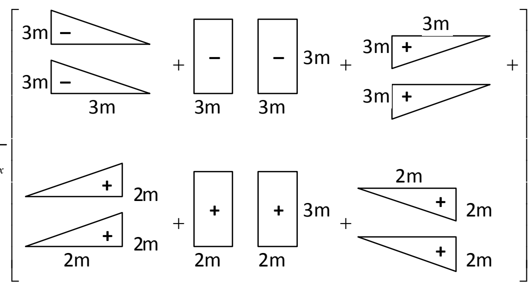
$$\delta_{11} = \frac{1}{E \cdot I_x}$$



$$\delta_{11} = \frac{1}{E \cdot I_x}\left(\frac{1}{2}\cdot 1 \cdot 7m \cdot \frac{2}{3}\cdot 1\right) \qquad \delta_{11} = 5.319 \times 10^{-4}\frac{1}{kN \cdot m} \qquad E \cdot I_x \delta_{11} = 2.333\, m$$

$$\delta_{11} = 5.319 \times 10^{-4} \cdot \frac{1}{kN \cdot m}$$

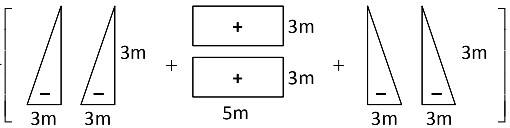
$$\delta_{22} = \frac{1}{E \cdot I_x}$$



$$\delta_{22} = \frac{1}{E \cdot I_x}\cdot\left(\frac{1}{2}\cdot 3 \cdot m \cdot 3 \cdot m \cdot \frac{2}{3}\cdot 3 \cdot m \cdot 2 + 3 \cdot m \cdot 3 \cdot m \cdot 3 \cdot m + \frac{1}{2}\cdot 2 \cdot m \cdot 2 \cdot m \cdot \frac{2}{3}\, 2m \cdot 2 + 2m \cdot 3m \cdot 2m\right)$$

$$\delta_{22} = 1.421 \times 10^{-2}\frac{\text{m}}{kN} \qquad E \cdot I_x \delta_{22} = 62.333\, m^3$$

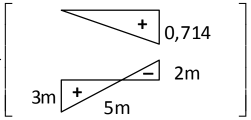
$$\delta_{22} = 1.421 \times 10^{-2}\frac{m}{kN}$$

$$\delta_{33} = \frac{1}{E \cdot I_x}\left[\right.$$


$$\left.\right]$$

$$\delta_{33} = \frac{1}{E \cdot I_x} \cdot \left(\frac{1}{2} \cdot 3 \cdot m \cdot 3 \cdot m \cdot \frac{2}{3} \cdot 3 \cdot m \cdot 2 + 3 \cdot m \cdot 5 \cdot m \cdot 3 \cdot m\right) \qquad \delta_{33} = 1.436 \times 10^{-2} \frac{\text{m}}{kN} \qquad E \cdot I_x \delta_{33} = 63\,m^3$$

$$\delta_{33} = 1.436 \times 10^{-2} \cdot \frac{m}{kN}$$

$$\delta_{12} = \frac{1}{E \cdot I_x}\left[\right.$$


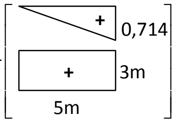
$$\left.\right]$$

$$\delta_{12} = \frac{1}{E \cdot I_x} \cdot \left[\frac{1}{2} \cdot 0.714 \cdot 5 \cdot m \cdot \left[\frac{2}{3} \cdot (-2 \cdot m) + \frac{1}{3} \cdot 3m\right]\right]$$

$$\delta_{12} = -1.356 \times 10^{-4} \frac{1}{kN} \qquad E \cdot I_x \delta_{12} = -0.595\,m^2$$

$$\delta_{12} = -1.356 \times 10^{-4} \cdot \frac{1}{kN}$$

$$\delta_{21} = \delta_{12}$$

$$\delta_{13} = \frac{1}{E \cdot I_x}\left[\right.$$


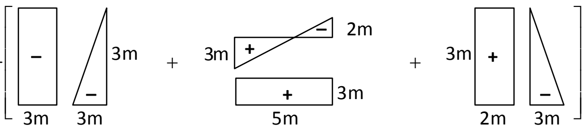
$$\left.\right]$$

$$\delta_{13} = \frac{1}{E \cdot I_x} \cdot \left(\frac{1}{2} \cdot 5 \cdot m \cdot 0.714 \cdot m \cdot 3m\right)$$

$$\delta_{13} = 1.221 \times 10^{-3}\,m\,\frac{1}{kN} \qquad E \cdot I_x \delta_{13} = 5.355\,m^3$$

$$\delta_{13} = 1.221 \times 10^{-3}\,m\,\frac{1}{kN}$$

$$\delta_{31} = \delta_{13}$$

$$\delta_{23} = \frac{1}{E \cdot I_x}\left[\right.$$


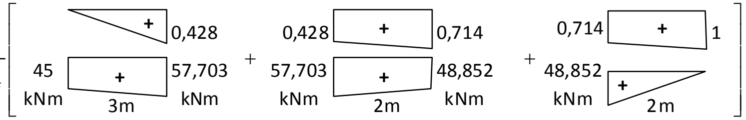
$$\left.\right]$$

$$\delta_{23} = \frac{1}{E \cdot I_x} \cdot \left[\frac{1}{2} \cdot 3 \cdot m \cdot (-3m) \cdot (-3m) + 5m \cdot 3m \cdot \frac{1}{2}[3m + (-2m)] + 2m \cdot 3m \cdot \frac{1}{2}(-3m)\right]$$

$$\delta_{23} = 2.735 \times 10^{-3} \frac{\text{m}}{kN} \qquad E \cdot I_x \delta_{23} = 12\,m^3$$

$$\delta_{23} = 2.735 \times 10^{-3} \frac{\text{m}}{kN} \qquad \delta_{32} = \delta_{23}$$

Wyznaczenie wyrazów wolnych układu równań

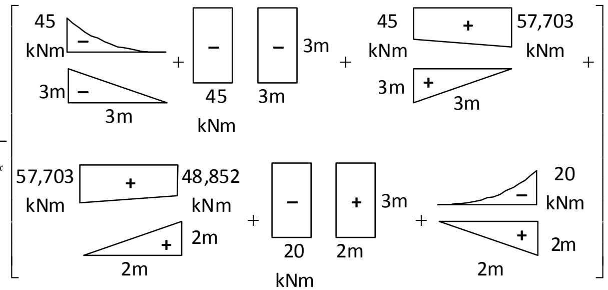
$$\Delta_{1p} = \frac{1}{E \cdot I_x}\left[\right.$$


$$\left.\right]$$

$$\Delta_{1p} = \frac{1}{E \cdot I_x}\left[\frac{1}{2} \cdot 3m \cdot 0.428 \cdot \left(\frac{1}{3} 45kN \cdot m + \frac{2}{3} \cdot 57.703kN \cdot m\right) + \frac{1}{2} \cdot 0.428 \cdot 2m \cdot \left(\frac{2}{3} 57.703kN \cdot m + \frac{1}{3} \cdot 48.852kN \cdot m\right) \ldots \right.$$
$$\left. + \frac{1}{2} \cdot 0.714 \cdot 2m \cdot \left(\frac{1}{3} 57.703kN \cdot m + \frac{2}{3} \cdot 48.852kN \cdot m\right) + \frac{1}{2} \cdot 2m \cdot 48.852kN \cdot m \cdot \left(\frac{2}{3} 0.714 + \frac{1}{3} \cdot 1\right)\right]$$

$$\Delta_{1p} = 3.061 \times 10^{-2} \qquad E \cdot I_x \Delta_{1p} = 134.285\,kN \cdot m^2$$

$$\Delta_{1p} = 3.061 \times 10^{-2}$$

$$\Delta_{2p} = \frac{1}{E \cdot I_x}\left[\right.$$


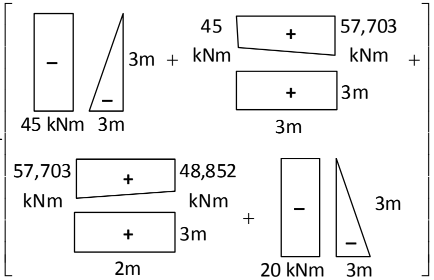
$$\left.\right]$$

$$\Delta_{2p} = \frac{1}{E \cdot I_x}\left[\frac{1}{3}(-45kN \cdot m) \cdot 3m \cdot \frac{3}{4}(-3m) + (-45kN \cdot m) \cdot 3m \cdot (-3m) + \frac{1}{2} \cdot 3m \cdot 3m \cdot \left(\frac{2}{3} 45kN \cdot m + \frac{1}{3} \cdot 57.703kN \cdot m\right) \ldots \right.$$
$$\left. + \frac{1}{2} \cdot 2m \cdot (-2m) \cdot \left(\frac{2}{3} 48.852kN \cdot m + \frac{1}{3} \cdot 57.703kN \cdot m\right) + (-20kN \cdot m) \cdot 2m \cdot 3m + \frac{1}{3} \cdot (-20kN \cdot m) \cdot 2m \cdot \frac{3}{4} \cdot 2m\right]$$

$$\Delta_{2p} = 1.104 \times 10^{-1}\,m \qquad E \cdot I_x \Delta_{2p} = 484.2\,kN \cdot m^3$$

$$\Delta_{2p} = 1.104 \times 10^{-1}\,m$$

$$\Delta_{3p} = \frac{1}{E \cdot I_x}\left[\ \cdots\ \right]$$



$$\Delta_{3p} = \frac{1}{E \cdot I_x}\cdot\left[\begin{aligned}&(-45 \cdot kN \cdot m)\cdot 3\cdot m\cdot\frac{1}{2}\cdot(-3\cdot m) + 3\cdot m\cdot 3\cdot m\cdot\frac{1}{2}\cdot(45\cdot kN\cdot m + 57.703\cdot kN\cdot m)\ \ldots\\ &+ 3\cdot m\cdot 2\cdot m\cdot\frac{1}{2}\cdot(48.852\cdot kN\cdot m + 57.703\cdot kN\cdot m) + 3\cdot m\cdot(-20\cdot kN\cdot m)\cdot\frac{1}{2}\cdot(-3\cdot m)\end{aligned}\right]$$

$\Delta_{3p} = 2.449 \times 10^{-1}\, m$ $\qquad$ $E \cdot I_x \Delta_{3p} = 1.074 \times 10^{3}\, kN \cdot m^{3}$

$\Delta_{3p} = 2.449 \times 10^{-1}\, m$

$\Delta_{1\Delta} = -[0.143 \cdot (-0.01)]$ $\qquad$ $\Delta_{1\Delta} = 1.43 \times 10^{-3}$

$\Delta_{2\Delta} = 0$

$\Delta_{3\Delta} = 0$

$\Delta_{1t0} = 0$

$\Delta_{2t0} = \alpha_t \cdot 10K \cdot (-1) \cdot 3m$ $\qquad$ $\Delta_{2t0} = -3.6 \times 10^{-4}\, m$

$\Delta_{3t0} = \alpha_t \cdot 20K \cdot (-1) \cdot 5m$ $\qquad$ $\Delta_{3t0} = -1.2 \times 10^{-3}\, m$

$\Delta_{1\Delta t} = \frac{\alpha_t}{h} \cdot 20K \cdot \frac{1}{2} \cdot 5m \cdot 0.714$ $\qquad$ $\Delta_{1\Delta t} = 2.142 \times 10^{-3}$ $\qquad$ $\Delta_{1\Delta t} = 2.142 \times 10^{-3}$

$\Delta_{2\Delta t} = \frac{\alpha_t}{h} \cdot 20K \cdot \left[\frac{1}{2} \cdot 3m \cdot 3m + \frac{1}{2} \cdot 2m \cdot (-2m)\right]$ $\qquad$ $\Delta_{2\Delta t} = 3 \times 10^{-3}\, m$

$\Delta_{3\Delta t} = \frac{\alpha_t}{h} \cdot 20K \cdot 3m \cdot 5m$ $\qquad$ $\Delta_{3\Delta t} = 0.018\, m$

$\Delta_{10} = \Delta_{1p} + \left(\Delta_{1\Delta} + \Delta_{1t0} + \Delta_{1\Delta t}\right)$ $\qquad$ $\Delta_{10} = 0.034$

$\Delta_{20} = \Delta_{2p} + \left(\Delta_{2\Delta} + \Delta_{2t0} + \Delta_{2\Delta t}\right)$ $\qquad$ $\Delta_{20} = 0.113\, m$

$\Delta_{30} = \Delta_{3p} + \left(\Delta_{3\Delta} + \Delta_{3t0} + \Delta_{3\Delta t}\right)$ $\qquad$ $\Delta_{30} = 0.262\, m$

Rozwiązanie układu równań

$\delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 + \delta_{13} \cdot X_3 + \Delta_{10} = 0.01$

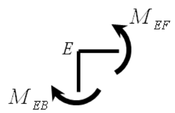
$\delta_{21} \cdot X_1 + \delta_{22} \cdot X_2 + \delta_{23} \cdot X_3 + \Delta_{20} = 0$

$\delta_{31} \cdot X_1 + \delta_{32} \cdot X_2 + \delta_{33} \cdot X_3 + \Delta_{30} = 0$

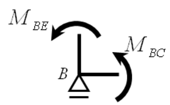
$X_1 = -8.68\, kN \cdot m$ $\qquad$ $X_2 = -4.85\, kN$ $\qquad$ $X_3 = -16.562\, kN$

X1	-8.68				
X2	-4.85				
X3	-16.562				
	Mp	**M1**	**M2**	**M3**	**Most**
AD	0	1	0	0	**-8.680**
DA	48.852	0.714	0	0	**42.654**
DC	48.852	0.714	-2	3	**2.668**
DG	-20	0	2	-3	**19.986**
CD	57.703	0.428	0	3	**4.302**
CB	57.703	0.428	0	3	**4.302**
BC	45	0	3	3	**-19.236**
BE	-45	0	-3	-3	**19.236**
EB	-45	0	-3	0	**-30.450**
EF	-45	0	-3	0	**-30.450**
FE	0	0	0	0	**0.000**
FG	0	0	0	0	**0.000**
GF	-20	0	2	0	**-29.700**
GD	-20	0	2	0	**-29.700**

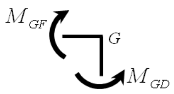
Sprawdzenie równowagi momentów w węzłach



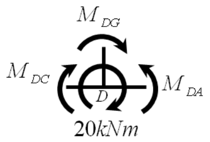
$M_{EB} - M_{EF} = 0\, kN \cdot m$



$M_{BE} + M_{BC} = 0\, kN \cdot m$



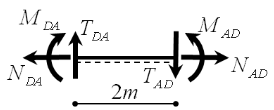
$M_{GF} - M_{GD} = 0\,kN \cdot m$



$M_{DC} + M_{DG} - M_{DA} + 20kN \cdot m = 0\,kN \cdot m$

Wyznaczenie sił tnących

Pręt DA



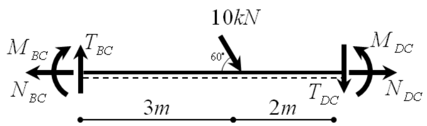
ΣM$_A$=0

$T_{DA} \cdot 2m + M_{DA} - M_{AD} = 0$

ΣY=0 $T_{AD} = T_{DA}$

$T_{DA} = -25.667\,kN$ $T_{AD} = -25.667\,kN$

Pręt BD



ΣM$_C$=0

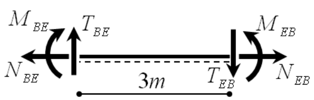
$T_{BC} \cdot 5m + M_{BC} - M_{DC} - 10kN \cdot sin(60deg) \cdot 2m = 0$

ΣY=0

$T_{BC} - T_{DC} - 10kN \cdot sin(60deg) = 0$

$T_{DC} = -0.815\,kN$ $T_{BC} = 7.845\,kN$

Pręt BE



ΣM$_E$=0

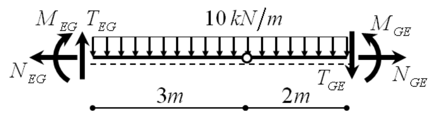
$T_{BE} \cdot 3m + M_{BE} - M_{EB} = 0$

ΣY=0

$T_{EB} = T_{BE}$

$T_{BE} = -16.562\,kN$ $T_{EB} = -16.562\,kN$

Pręt EG



ΣM$_G$=0

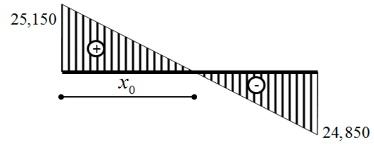
$T_{EF} \cdot 5m + M_{EF} - M_{GF} - 10\frac{kN}{m} \cdot 5m \cdot \frac{5m}{2} = 0$

ΣY=0

$T_{EF} - T_{GF} - 10\frac{kN}{m} \cdot 5m = 0$

$T_{GF} = -24.85\,kN$ $T_{EF} = 25.15\,kN$

Wyznaczenie maksymalnego momentu

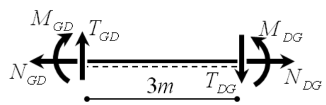


$\frac{T_{EF}}{x_0} = \frac{-T_{GF}}{5m - x_0}$

$x_0 = 2.515\,m$

$M_{max} = M_{EF} + T_{EF} \cdot x_0 - 10\frac{kN}{m} \cdot x_0 \cdot \frac{x_0}{2}$ $M_{max} = 1.176\,kN \cdot m$

Pręt GD



ΣM$_D$=0

$T_{GD} \cdot 3m + M_{GD} - M_{DG} = 0$

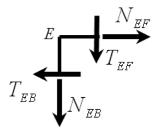
ΣY=0

$T_{DG} = T_{GD}$

$T_{GD} = 16.562\,kN$ $T_{DG} = 16.562\,kN$

Równowaga węzłów - siły normalne i reakcje

Węzeł E



ΣX=0 $N_{EF} - T_{EB} = 0$

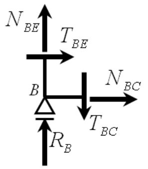
$N_{EF} = -16.562\,kN$

ΣY=0 $N_{EB} + T_{EF} = 0$

$N_{EB} = -25.15\,kN$

$N_{GF} = N_{EF}$ $N_{BE} = N_{EB}$ (z warunków równowagi pretów EG i BE)

Węzeł B



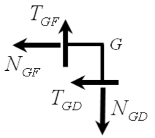
ΣX=0 $N_{BC} + T_{BE} = 0$

$N_{BC} = 16.562\,kN$

ΣY=0 $N_{BE} + R_B - T_{BC} = 0$

$R_B = 32.995\,kN$

Węzeł G



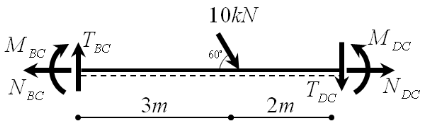
ΣX=0 $T_{GD} + N_{GF} = 0\,kN$ (sprawdzenie)

ΣY=0 $-N_{GD} + T_{GF} = 0$

$N_{GD} = -24.85\,kN$

$N_{DG} = N_{GD}$ (z warunku równowagi preta DG)

Pręt BD

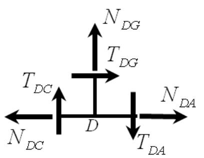


ΣX=0

$N_{DC} + 10kN\cdot cos(60deg) - N_{BC} = 0$

$N_{DC} = 11.562\,kN$

Węzeł D



ΣY=0

$N_{DG} + T_{DC} - T_{DA} = 1.648 \times 10^{-3}\,kN$ (sprawdzenie)

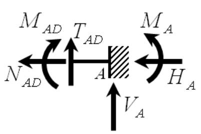
ΣX=0

$N_{DA} + T_{DG} - N_{DC} = 0$

$N_{DA} = -5\,kN$

$N_{AD} = N_{DA}$ (z warunku równowagi preta DA)

Węzeł A



ΣM_A=0 $M_A = M_{AD}$ $M_A = -8.68\,kN\cdot m$

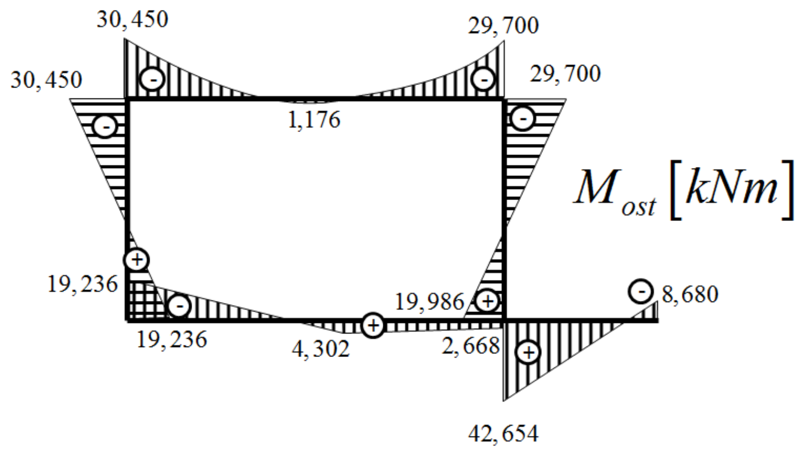
ΣX=0 $H_A + N_{AD} = 0$

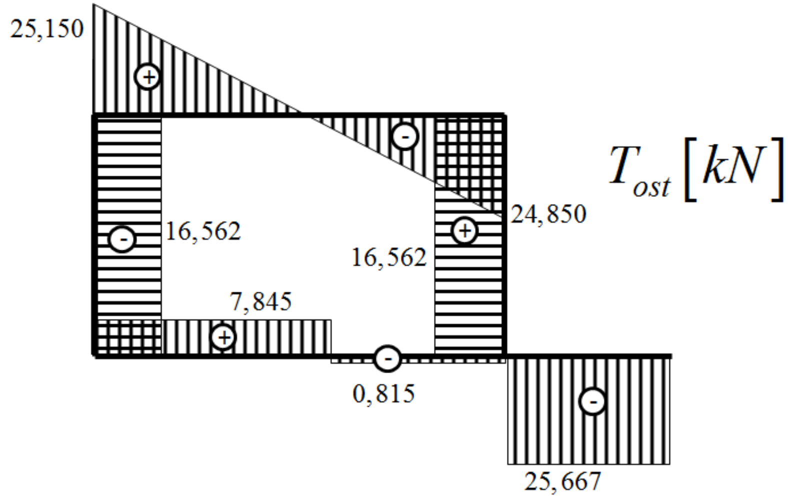
$H_A = 5\,kN$

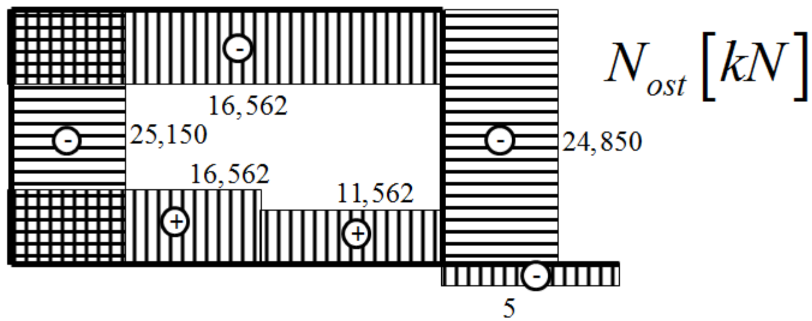
ΣY=0 $V_A + T_{AD} = 0$

$V_A = 25.667\,kN$

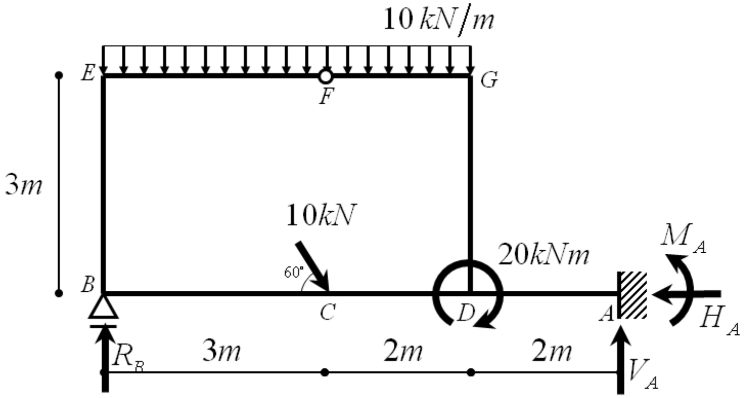
Ostateczne wykresy sił wewnętrznych







Sprawdzenia statyczne



ΣX=0

$$-H_A + 10kN \cdot cos(60deg) = 0\,kN$$

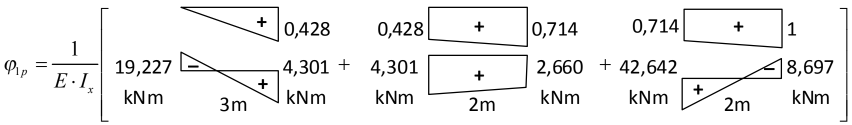
ΣY=0

$$R_B + V_A - 10kN\,sin(60deg) - 10\,\frac{kN}{m} \cdot 5m = 1.648 \times 10^{-3}\,kN$$

ΣM_G=0

$$R_B \cdot 5m - 10\,\frac{kN}{m} \cdot 5m \cdot \frac{5m}{2} - 10kN \cdot sin(60deg) \cdot 2m - 10kN \cdot cos(60deg) \cdot 3m \;... = 1.295 \times 10^{-12}\,kN \cdot m$$
$$+ 20kN \cdot m - V_A \cdot 2m + H_A \cdot 3m - M_A$$

Sprawdzenie kinematyczne



$$\varphi_{1p} = \frac{1}{E \cdot I_x}\left[\frac{1}{2} \cdot 3m \cdot 0.428 \cdot \left(\frac{1}{3} \cdot M_{BC} + \frac{2}{3} \cdot M_{CB}\right) ... + \frac{1}{2} \cdot 2m \cdot M_{CD} \cdot \left(\frac{2}{3} \cdot 0.428 + \frac{1}{3} \cdot 0.714\right) + \frac{1}{2} \cdot 2m \cdot M_{DC} \cdot \left(\frac{1}{3} \cdot 0.428 + \frac{2}{3} \cdot 0.714\right) ... + \frac{1}{2} \cdot 2m \cdot M_{DA} \cdot \left(\frac{2}{3} \cdot 0.714 + \frac{1}{3} \cdot 1\right) + \frac{1}{2} \cdot 2m \cdot M_{AD} \cdot \left(\frac{1}{3} \cdot 0.714 + \frac{2}{3} \cdot 1\right)\right]$$

$$\varphi_{1p} = 6.45 \times 10^{-3}$$

$\varphi_{1t} = \Delta_{1\Delta t}$ $\qquad \varphi_{1t} = 2.142 \times 10^{-3}$

$\varphi_{1\Delta} = -(1 \cdot 0.01 - 0.143 \cdot 0.01)$ $\qquad \varphi_{1\Delta} = -8.57 \times 10^{-3}$

$\varphi_1 = \varphi_{1p} + \varphi_{1t} + \varphi_{1\Delta}$ $\qquad \varphi_1 = 2.186 \times 10^{-5}$

$\Delta = \dfrac{\varphi_1}{\Delta_{1p}}$ $\qquad \Delta = 0.071\,\%$