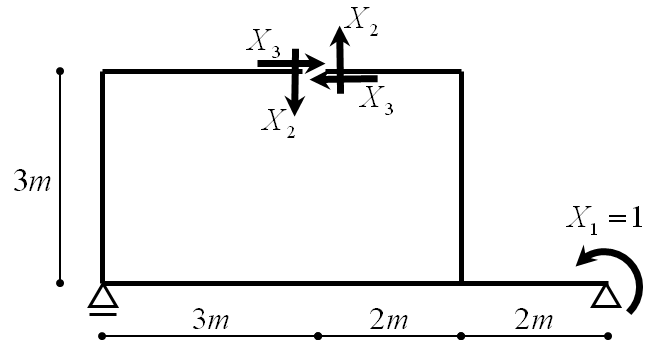
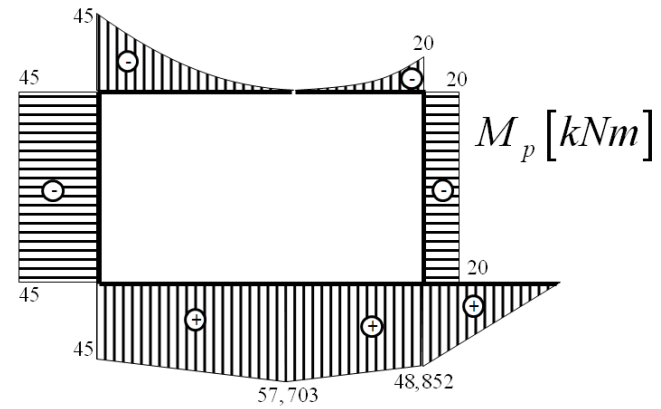
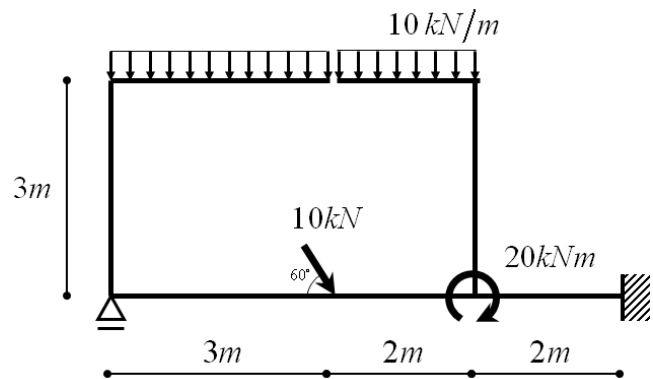


$E = 205 \text{ GPa}$ $\alpha_t = 1.2 \cdot 10^{-5} \cdot \frac{1}{\text{K}}$
 Przekrój: I200 $h = 0.2 \text{ m}$ $I_x = 2140 \text{ cm}^4$ $E \cdot I_x = 4387 \text{ kN} \cdot \text{m}^2$

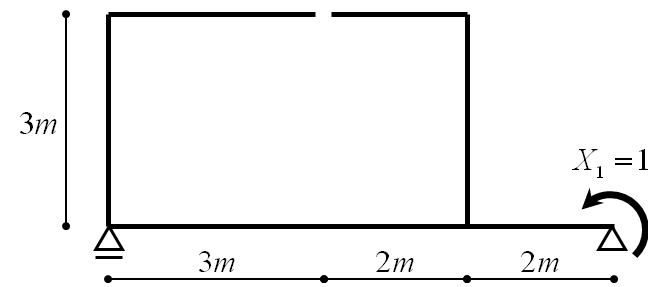
Układ podstawowy metody sił (UPMS)



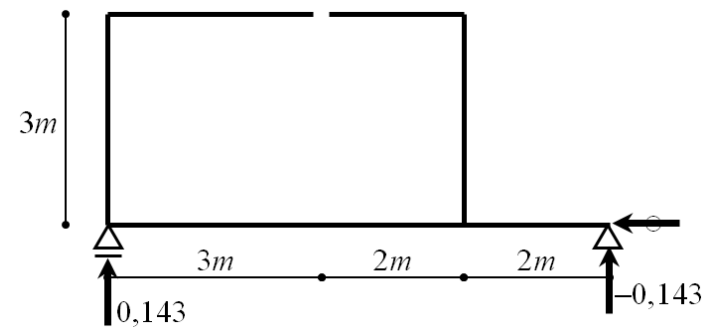
Stan "p"

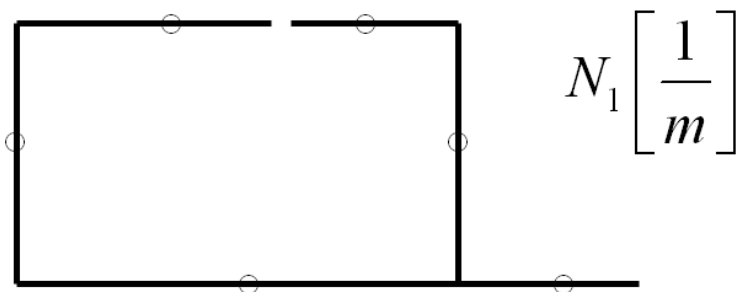
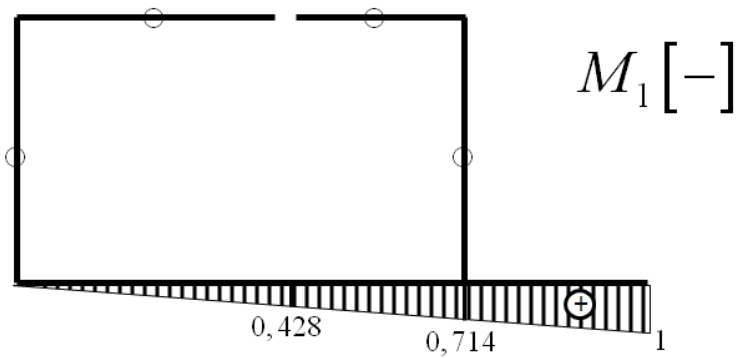


Stan X1=1

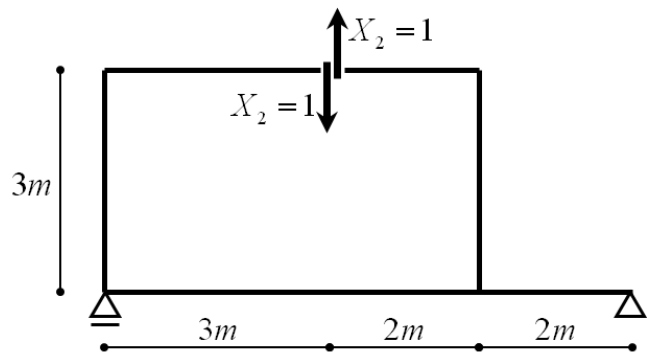


Reakcje w stanie X1=1

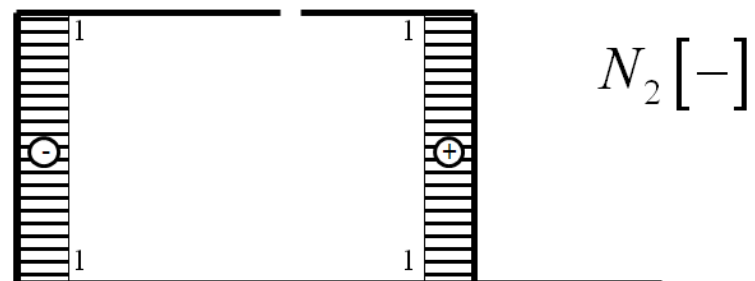
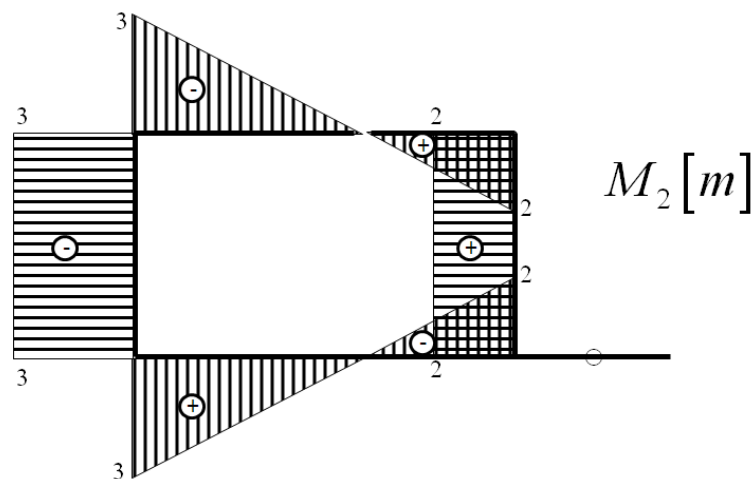
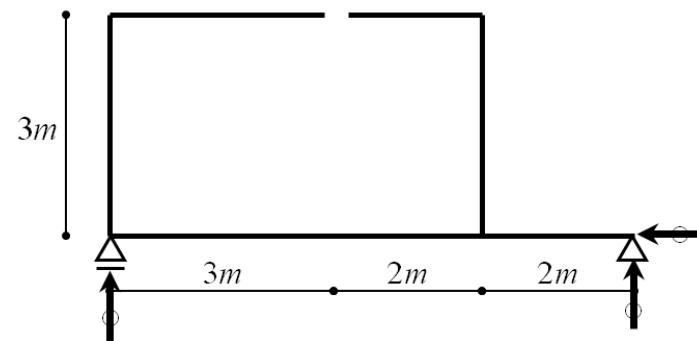




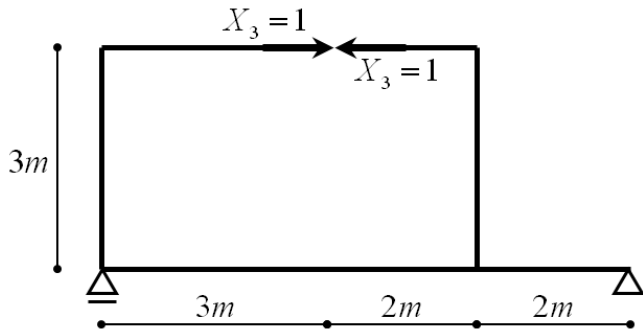
Stan $X_2=1$



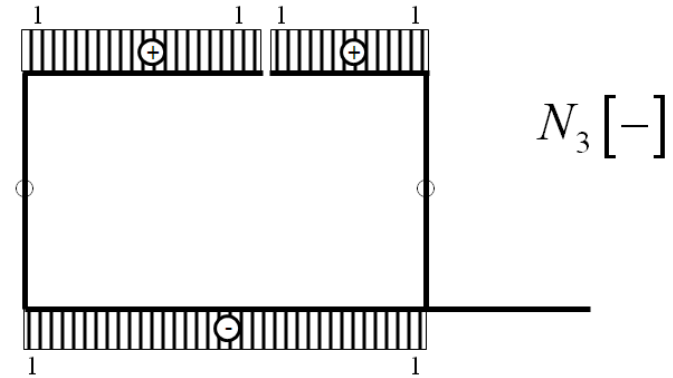
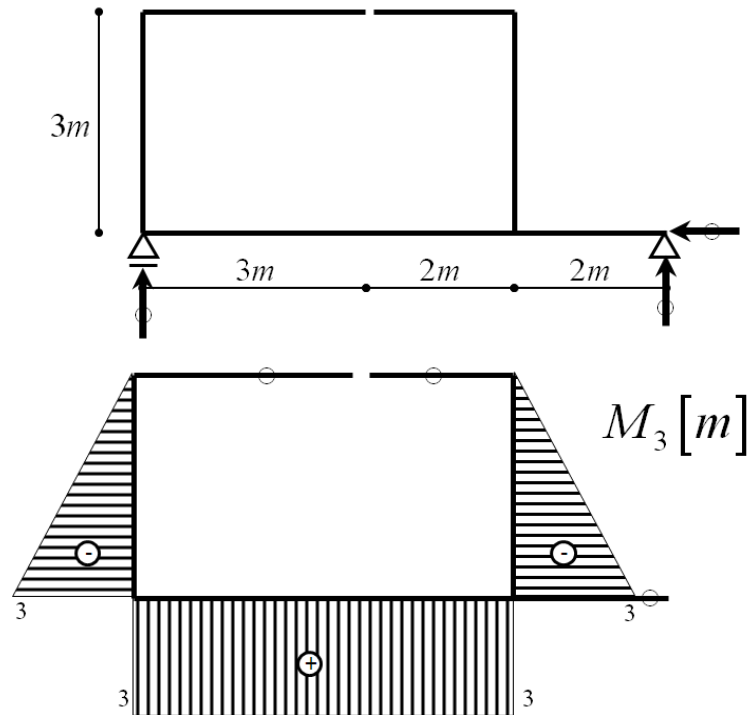
Reakcje w stanie $X_2=1$



Stan $X_3=1$



Reakcje w stanie $X_3=1$



Wyznaczenie współczynników układu równań

$$\delta_{11} = \frac{1}{E \cdot I_x} \left[\begin{array}{c} \text{7m} \\ \text{+} \\ \text{1} \\ \text{-} \\ \text{+} \\ \text{1} \end{array} \right]$$

$$\delta_{11} = \frac{1}{E \cdot I_x} \left(\frac{1}{2} \cdot 1 \cdot 7m \cdot \frac{2}{3} \cdot 1 \right) \quad \delta_{11} = 5.319 \times 10^{-4} \frac{1}{kN \cdot m} \quad E \cdot I_x \delta_{11} = 2.333 m$$

$$\delta_{11} = 5.319 \times 10^{-4} \frac{1}{kN \cdot m}$$

$$\delta_{22} = \frac{1}{E \cdot I_x} \left[\begin{array}{c} \text{3m} \text{ -} \\ \text{3m} \text{ -} \\ \text{3m} \\ \text{+} \\ \text{2m} \\ \text{+} \\ \text{2m} \\ \text{+} \\ \text{2m} \\ \text{+} \\ \text{2m} \\ \text{+} \\ \text{3m} \\ \text{+} \\ \text{2m} \\ \text{+} \\ \text{2m} \end{array} \right]$$

$$\delta_{22} = \frac{1}{E \cdot I_x} \left(\frac{1}{2} \cdot 3 \cdot m \cdot 3 \cdot m \cdot \frac{2}{3} \cdot 3 \cdot m \cdot 2 + 3 \cdot m \cdot 3 \cdot m \cdot 3 \cdot m + \frac{1}{2} \cdot 2 \cdot m \cdot 2 \cdot m \cdot \frac{2}{3} \cdot 2m \cdot 2 + 2m \cdot 3m \cdot 2m \right)$$

$$\delta_{22} = 1.421 \times 10^{-2} \frac{m}{kN} \quad E \cdot I_x \delta_{22} = 62.333 m^3$$

$$\delta_{22} = 1.421 \times 10^{-2} \frac{m}{kN}$$

$$\delta_{33} = \frac{1}{E \cdot I_x} \left[\begin{array}{c} \text{Diagram 1: Two triangles, each with base 3m and height 3m, one positive and one negative.} \\ \text{Diagram 2: Two rectangles, each with width 3m and height 3m, both positive.} \\ \text{Diagram 3: Two triangles, each with base 3m and height 3m, one positive and one negative.} \end{array} \right]$$

$$\delta_{33} = \frac{1}{E \cdot I_x} \cdot \left(\frac{1}{2} \cdot 3 \cdot 3 \cdot 3 \cdot m \cdot \frac{2}{3} \cdot 3 \cdot 3 \cdot 2 + 3 \cdot 3 \cdot 5 \cdot 3 \cdot m \right) \quad \delta_{33} = 1.436 \times 10^{-2} \frac{m}{kN} \quad E \cdot I_x \delta_{33} = 63 m^3$$

$$\delta_{33} = 1.436 \times 10^{-2} \frac{m}{kN}$$

$$\delta_{12} = \frac{1}{E \cdot I_x} \left[\begin{array}{c} \text{Diagram 1: Triangle with base 3m, height 0.714, positive.} \\ \text{Diagram 2: Triangle with base 5m, height 2m, negative.} \end{array} \right] \quad \delta_{12} = \frac{1}{E \cdot I_x} \cdot \left[\frac{1}{2} \cdot 0.714 \cdot 5 \cdot m \cdot \left[\frac{2}{3} \cdot (-2 \cdot m) + \frac{1}{3} \cdot 3m \right] \right]$$

$$\delta_{12} = -1.356 \times 10^{-4} \frac{1}{kN} \quad E \cdot I_x \delta_{12} = -0.595 m^2$$

$$\delta_{12} = -1.356 \times 10^{-4} \frac{1}{kN}$$

$$\delta_{21} = \delta_{12}$$

$$\delta_{13} = \frac{1}{E \cdot I_x} \left[\begin{array}{c} \text{Diagram 1: Triangle with base 3m, height 0.714, positive.} \\ \text{Diagram 2: Rectangle with width 5m, height 3m, positive.} \end{array} \right] \quad \delta_{13} = \frac{1}{E \cdot I_x} \cdot \left(\frac{1}{2} \cdot 5 \cdot m \cdot 0.714 \cdot m \cdot 3m \right)$$

$$\delta_{13} = 1.221 \times 10^{-3} m \frac{1}{kN} \quad E \cdot I_x \delta_{13} = 5.355 m^3$$

$$\delta_{13} = 1.221 \times 10^{-3} m \frac{1}{kN}$$

$$\delta_{31} = \delta_{13}$$

$$\delta_{23} = \frac{1}{E \cdot I_x} \left[\begin{array}{c} \text{Diagram 1: Rectangle with width 3m, height 3m, negative.} \\ \text{Diagram 2: Triangle with base 3m, height 2m, positive.} \\ \text{Diagram 3: Rectangle with width 5m, height 3m, positive.} \\ \text{Diagram 4: Triangle with base 2m, height 3m, negative.} \end{array} \right]$$

$$\delta_{23} = \frac{1}{E \cdot I_x} \cdot \left[\frac{1}{2} \cdot 3 \cdot m \cdot (-3m) \cdot (-3m) + 5m \cdot 3m \cdot \frac{1}{2} [3m + (-2m)] + 2m \cdot 3m \cdot \frac{1}{2} \cdot (-3m) \right]$$

$$\delta_{23} = 2.735 \times 10^{-3} \frac{m}{kN} \quad E \cdot I_x \delta_{23} = 12 m^3$$

$$\delta_{23} = 2.735 \times 10^{-3} \frac{m}{kN}$$

$$\delta_{32} = \delta_{23}$$

Wyznaczenie wyrazów wolnych układu równań

$$\Delta_{1p} = \frac{1}{E \cdot I_x} \left[\begin{array}{c} \text{Diagram 1: Triangle with base 3m, height 0.428, positive.} \\ \text{Diagram 2: Rectangle with width 3m, height 57,703, positive.} \\ \text{Diagram 3: Rectangle with width 2m, height 48,852, positive.} \\ \text{Diagram 4: Triangle with base 2m, height 48,852, positive.} \end{array} \right]$$

$$\Delta_{1p} = \frac{1}{E \cdot I_x} \left[\frac{1}{2} \cdot 3m \cdot 0.428 \cdot \left(\frac{1}{3} \cdot 45kN \cdot m + \frac{2}{3} \cdot 57.703kN \cdot m \right) + \frac{1}{2} \cdot 0.428 \cdot 2m \cdot \left(\frac{2}{3} \cdot 57.703kN \cdot m + \frac{1}{3} \cdot 48.852kN \cdot m \right) \dots \right]$$

$$\Delta_{1p} = 3.061 \times 10^{-2} \quad E \cdot I_x \Delta_{1p} = 134.285 kN \cdot m^2$$

$$\Delta_{1p} = 3.061 \times 10^{-2}$$

$$\Delta_{2p} = \frac{1}{E \cdot I_x} \left[\begin{array}{c} \text{Diagram 1: Triangle with base 3m, height 45, negative.} \\ \text{Diagram 2: Rectangle with width 3m, height 45, negative.} \\ \text{Diagram 3: Triangle with base 3m, height 45, positive.} \\ \text{Diagram 4: Rectangle with width 3m, height 57,703, positive.} \\ \text{Diagram 5: Triangle with base 2m, height 57,703, positive.} \\ \text{Diagram 6: Rectangle with width 2m, height 48,852, positive.} \\ \text{Diagram 7: Rectangle with width 2m, height 20, positive.} \\ \text{Diagram 8: Triangle with base 2m, height 20, negative.} \end{array} \right]$$

$$\Delta_{2p} = \frac{1}{E \cdot I_x} \left[\frac{1}{3} \cdot (-45kN \cdot m) \cdot 3m \cdot \frac{3}{4} \cdot (-3m) + (-45kN \cdot m) \cdot 3m \cdot (-3m) + \frac{1}{2} \cdot 3m \cdot 3m \cdot \left(\frac{2}{3} \cdot 45kN \cdot m + \frac{1}{3} \cdot 57.703kN \cdot m \right) \dots \right]$$

$$\Delta_{2p} = 1.104 \times 10^{-1} m \quad E \cdot I_x \Delta_{2p} = 484.2 kN \cdot m^3$$

$$\Delta_{2p} = 1.104 \times 10^{-1} m$$

$$\Delta_{3p} = \frac{1}{E \cdot I_x} \left[\begin{array}{l} \begin{array}{l} \text{45 kNm} \cdot 3\text{m} \\ \text{3m} \end{array} + \begin{array}{l} \text{45 kNm} \\ \text{3m} \end{array} \cdot \begin{array}{l} \text{57,703 kNm} \\ \text{3m} \end{array} + \\ \begin{array}{l} \text{57,703 kNm} \\ \text{3m} \end{array} \cdot \begin{array}{l} \text{48,852 kNm} \\ \text{3m} \end{array} + \begin{array}{l} \text{20 kNm} \\ \text{3m} \end{array} \end{array} \right]$$

$$\Delta_{3p} = \frac{1}{E \cdot I_x} \left[(-45 \cdot \text{kN} \cdot \text{m}) \cdot 3 \cdot \text{m} \cdot \frac{1}{2} \cdot (-3 \cdot \text{m}) + 3 \cdot \text{m} \cdot 3 \cdot \text{m} \cdot \frac{1}{2} \cdot (45 \cdot \text{kN} \cdot \text{m} + 57.703 \cdot \text{kN} \cdot \text{m}) \dots \right]$$

$$\Delta_{3p} = 2.449 \times 10^{-1} \text{ m} \quad E \cdot I_x \Delta_{3p} = 1.074 \times 10^3 \text{ kN} \cdot \text{m}^3$$

$$\Delta_{3p} = 2.449 \times 10^{-1} \text{ m}$$

$$\Delta_{1A} = -[0.143 \cdot (-0.01)] \quad \Delta_{1A} = 1.43 \times 10^{-3}$$

$$\Delta_{2A} = 0$$

$$\Delta_{3A} = 0$$

$$\Delta_{10} = 0$$

$$\Delta_{20} = \alpha_f \cdot 10 \text{K} \cdot (-1) \cdot 3 \text{m} \quad \Delta_{20} = -3.6 \times 10^{-4} \text{ m}$$

$$\Delta_{30} = \alpha_f \cdot 20 \text{K} \cdot (-1) \cdot 5 \text{m} \quad \Delta_{30} = -1.2 \times 10^{-3} \text{ m}$$

$$\Delta_{1A} = \frac{\alpha_t}{h} \cdot 20 \text{K} \cdot \frac{1}{2} \cdot 5 \text{m} \cdot 0.714 \quad \Delta_{1A} = 2.142 \times 10^{-3} \quad \Delta_{1A} = 2.142 \times 10^{-3}$$

$$\Delta_{2A} = \frac{\alpha_t}{h} \cdot 20 \text{K} \cdot \left[\frac{1}{2} \cdot 3 \text{m} \cdot 3 \text{m} + \frac{1}{2} \cdot 2 \text{m} \cdot (-2 \text{m}) \right] \quad \Delta_{2A} = 3 \times 10^{-3} \text{ m}$$

$$\Delta_{3A} = \frac{\alpha_t}{h} \cdot 20 \text{K} \cdot 3 \text{m} \cdot 5 \text{m} \quad \Delta_{3A} = 0.018 \text{ m}$$

$$\Delta_{10} = \Delta_{1p} + (\Delta_{1A} + \Delta_{10} + \Delta_{1A}) \quad \Delta_{10} = 0.034$$

$$\Delta_{20} = \Delta_{2p} + (\Delta_{2A} + \Delta_{20} + \Delta_{2A}) \quad \Delta_{20} = 0.113 \text{ m}$$

$$\Delta_{30} = \Delta_{3p} + (\Delta_{3A} + \Delta_{30} + \Delta_{3A}) \quad \Delta_{30} = 0.262 \text{ m}$$

Rozwiązanie układu równań

$$\delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 + \delta_{13} \cdot X_3 + \Delta_{10} = 0.01$$

$$\delta_{21} \cdot X_1 + \delta_{22} \cdot X_2 + \delta_{23} \cdot X_3 + \Delta_{20} = 0$$

$$\delta_{31} \cdot X_1 + \delta_{32} \cdot X_2 + \delta_{33} \cdot X_3 + \Delta_{30} = 0$$

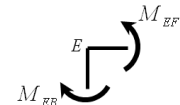
$$X_1 = -8.68 \text{ kN} \cdot \text{m}$$

$$X_2 = -4.85 \text{ kN}$$

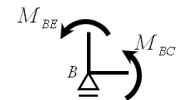
$$X_3 = -16.562 \text{ kN}$$

X1	-8.68				
X2	-4.85				
X3	-16.562				
	Mp	M1	M2	M3	Most
AD	0	1	0	0	-8.680
DA	48.852	0.714	0	0	42.654
DC	48.852	0.714	-2	3	2.668
DG	-20	0	2	-3	19.986
CD	57.703	0.428	0	3	4.302
CB	57.703	0.428	0	3	4.302
BC	45	0	3	3	-19.236
BE	-45	0	-3	-3	19.236
EB	-45	0	-3	0	-30.450
EF	-45	0	-3	0	-30.450
FE	0	0	0	0	0.000
FG	0	0	0	0	0.000
GF	-20	0	2	0	-29.700
GD	-20	0	2	0	-29.700

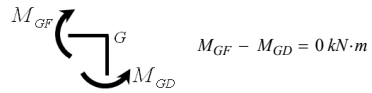
Sprawdzenie równowagi momentów w węzłach



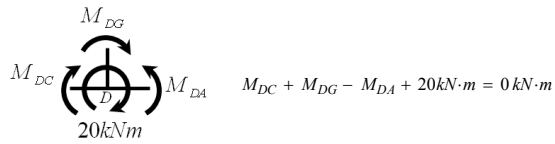
$$M_{EB} - M_{EF} = 0 \text{ kN} \cdot \text{m}$$



$$M_{BE} + M_{BC} = 0 \text{ kN} \cdot \text{m}$$



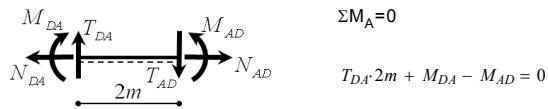
$$M_{GF} - M_{GD} = 0 \text{ kN}\cdot\text{m}$$



$$M_{DC} + M_{DG} - M_{DA} + 20 \text{ kN}\cdot\text{m} = 0 \text{ kN}\cdot\text{m}$$

Wyznaczenie sił tnących

Pręt DA



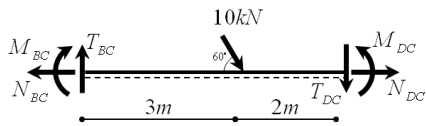
$$\Sigma M_A = 0$$

$$T_{DA} \cdot 2m + M_{DA} - M_{AD} = 0$$

$$\Sigma Y = 0 \quad T_{AD} = T_{DA}$$

$$T_{DA} = -25.667 \text{ kN} \quad T_{AD} = -25.667 \text{ kN}$$

Pręt BD



$$\Sigma M_C = 0$$

$$T_{BC} \cdot 5m + M_{BC} - M_{DC} - 10 \text{ kN} \cdot \sin(60 \text{ deg}) \cdot 2m = 0$$

$$\Sigma Y = 0$$

$$T_{BC} - T_{DC} - 10 \text{ kN} \cdot \sin(60 \text{ deg}) = 0$$

$$T_{DC} = -0.815 \text{ kN} \quad T_{BC} = 7.845 \text{ kN}$$

Pręt BE



$$\Sigma M_E = 0$$

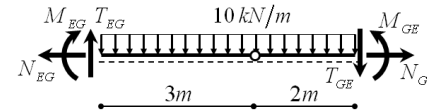
$$T_{BE} \cdot 3m + M_{BE} - M_{EB} = 0$$

$$\Sigma Y = 0$$

$$T_{EB} = T_{BE}$$

$$T_{BE} = -16.562 \text{ kN} \quad T_{EB} = -16.562 \text{ kN}$$

Pręt EG



$$\Sigma M_G = 0$$

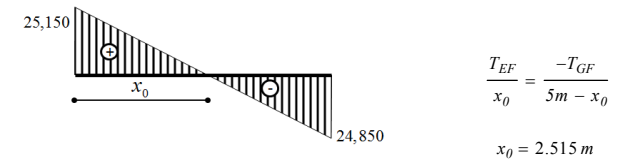
$$T_{EF} \cdot 5m + M_{EF} - M_{GF} - 10 \frac{\text{kN}}{\text{m}} \cdot 5m \cdot \frac{5m}{2} = 0$$

$$\Sigma Y = 0$$

$$T_{EF} - T_{GF} - 10 \frac{\text{kN}}{\text{m}} \cdot 5m = 0$$

$$T_{GF} = -24.85 \text{ kN} \quad T_{EF} = 25.15 \text{ kN}$$

Wyznaczenie maksymalnego momentu

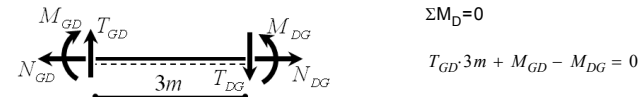


$$\frac{T_{EF}}{x_0} = \frac{-T_{GF}}{5m - x_0}$$

$$x_0 = 2.515 \text{ m}$$

$$M_{max} = M_{EF} + T_{EF} \cdot x_0 - 10 \frac{\text{kN}}{\text{m}} \cdot x_0 \cdot \frac{x_0}{2} \quad M_{max} = 1.176 \text{ kN}\cdot\text{m}$$

Pręt GD



$$\Sigma M_D = 0$$

$$T_{GD} \cdot 3m + M_{GD} - M_{DG} = 0$$

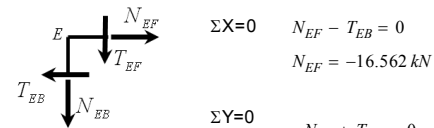
$$\Sigma Y = 0$$

$$T_{DG} = T_{GD}$$

$$T_{GD} = 16.562 \text{ kN} \quad T_{DG} = 16.562 \text{ kN}$$

Równowaga węzłów - siły normalne i reakcje

Węzeł E



$$\Sigma X = 0 \quad N_{EF} - T_{EB} = 0$$

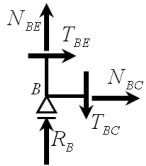
$$N_{EF} = -16.562 \text{ kN}$$

$$\Sigma Y = 0 \quad N_{EB} + T_{EF} = 0$$

$$N_{EB} = -25.15 \text{ kN}$$

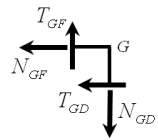
$$N_{GF} = N_{EF} \quad N_{BE} = N_{EB} \quad (\text{z warunków równowagi prętów EG i BE})$$

Węzeł B



$$\begin{aligned} \Sigma X=0 \quad N_{BC} + T_{BE} &= 0 \\ N_{BC} &= 16.562 \text{ kN} \\ \Sigma Y=0 \quad N_{BE} + R_B - T_{BC} &= 0 \\ R_B &= 32.995 \text{ kN} \end{aligned}$$

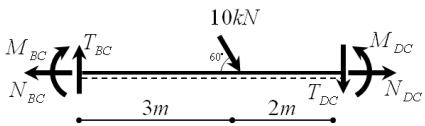
Węzeł G



$$\begin{aligned} \Sigma X=0 \quad T_{GD} + N_{GF} &= 0 \text{ kN} \quad (\text{sprawdzenie}) \\ \Sigma Y=0 \quad -N_{GD} + T_{GF} &= 0 \\ N_{GD} &= -24.85 \text{ kN} \end{aligned}$$

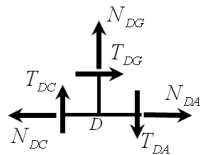
$$N_{DG} = N_{GD} \quad (\text{z warunku równowagi pręta DG})$$

Pręt BD



$$\begin{aligned} \Sigma X=0 \\ N_{DC} + 10 \text{ kN} \cdot \cos(60 \text{ deg}) - N_{BC} &= 0 \\ N_{DC} &= 11.562 \text{ kN} \end{aligned}$$

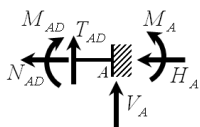
Węzeł D



$$\begin{aligned} \Sigma Y=0 \\ N_{DG} + T_{DC} - T_{DA} &= 1.648 \times 10^{-3} \text{ kN} \quad (\text{sprawdzenie}) \\ \Sigma X=0 \\ N_{DA} + T_{DG} - N_{DC} &= 0 \\ N_{DA} &= -5 \text{ kN} \end{aligned}$$

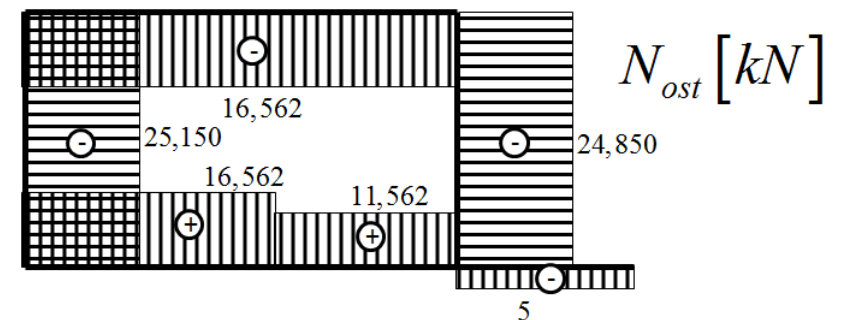
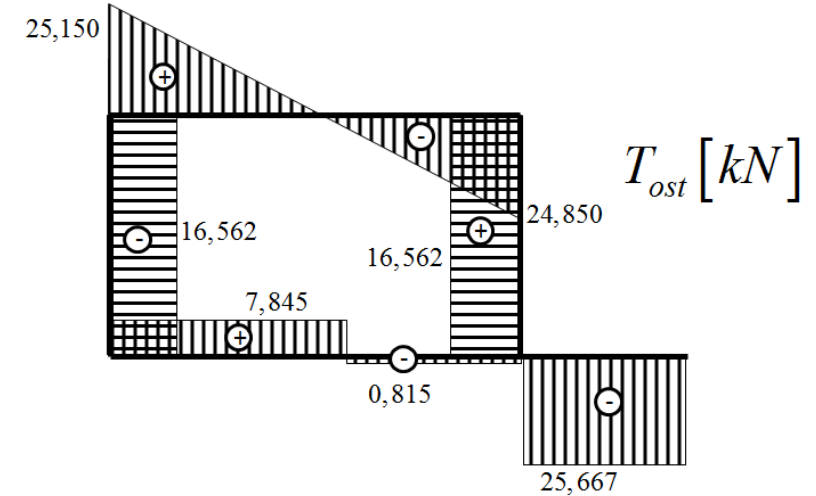
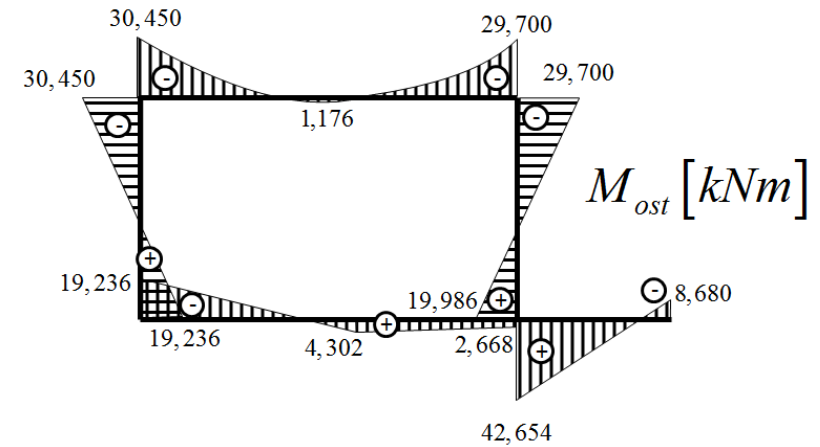
$$N_{AD} = N_{DA} \quad (\text{z warunku równowagi pręta DA})$$

Węzeł A

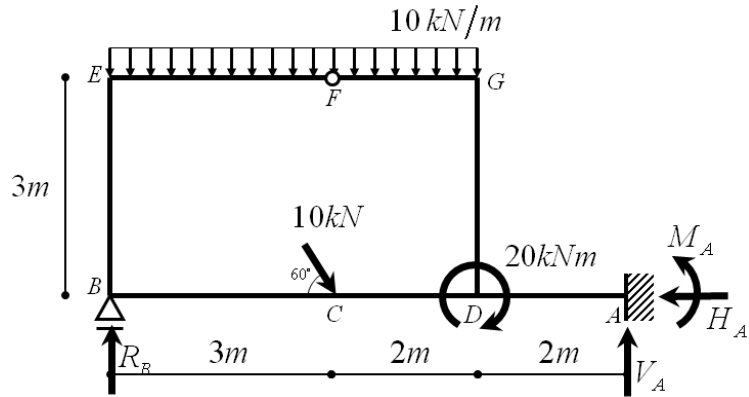


$$\begin{aligned} \Sigma M_A=0 \quad M_A &= M_{AD} \quad M_A = -8.68 \text{ kN}\cdot\text{m} \\ \Sigma X=0 \quad H_A + N_{AD} &= 0 \\ H_A &= 5 \text{ kN} \\ \Sigma Y=0 \quad V_A + T_{AD} &= 0 \\ V_A &= 25.667 \text{ kN} \end{aligned}$$

Ostateczne wykresy sił wewnętrznych



Sprawdzenia statyczne



$$\Sigma X=0$$

$$-H_A + 10 \text{ kN} \cdot \cos(60^\circ) = 0 \text{ kN}$$

$$\Sigma Y=0$$

$$R_B + V_A - 10 \text{ kN} \sin(60^\circ) - 10 \frac{\text{kN}}{\text{m}} \cdot 5 \text{ m} = 1.648 \times 10^{-3} \text{ kN}$$

$$\Sigma M_G=0$$

$$R_B \cdot 5 \text{ m} - 10 \frac{\text{kN}}{\text{m}} \cdot 5 \text{ m} \cdot \frac{5 \text{ m}}{2} - 10 \text{ kN} \cdot \sin(60^\circ) \cdot 2 \text{ m} - 10 \text{ kN} \cdot \cos(60^\circ) \cdot 3 \text{ m} \dots = 1.295 \times 10^{-12} \text{ kN} \cdot \text{m}$$

$$+ 20 \text{ kN} \cdot \text{m} - V_A \cdot 2 \text{ m} + H_A \cdot 3 \text{ m} - M_A$$

Sprawdzenie kinematyczne

$$\varphi_{1p} = \frac{1}{E \cdot I_x} \left[\begin{array}{cccc} \begin{array}{|c|} \hline \text{+} \\ \hline \end{array} 0,428 & \begin{array}{|c|} \hline \text{+} \\ \hline \end{array} 0,428 & \begin{array}{|c|} \hline \text{+} \\ \hline \end{array} 0,714 & \begin{array}{|c|} \hline \text{+} \\ \hline \end{array} 0,714 \\ \begin{array}{|c|} \hline \text{+} \\ \hline \end{array} 19,227 & \begin{array}{|c|} \hline \text{+} \\ \hline \end{array} 4,301 & \begin{array}{|c|} \hline \text{+} \\ \hline \end{array} 4,301 & \begin{array}{|c|} \hline \text{+} \\ \hline \end{array} 2,660 \\ \text{kNm} & \text{kNm} & \text{kNm} & \text{kNm} \\ \begin{array}{|c|} \hline \text{+} \\ \hline \end{array} 8,697 & & & \end{array} \right]$$

$$\varphi_{1p} = \frac{1}{E \cdot I_x} \left[\begin{array}{l} \frac{1}{2} \cdot 3 \text{ m} \cdot 0,428 \cdot \left(\frac{1}{3} \cdot M_{BC} + \frac{2}{3} \cdot M_{CB} \right) \dots \\ + \frac{1}{2} \cdot 2 \text{ m} \cdot M_{CD} \cdot \left(\frac{2}{3} \cdot 0,428 + \frac{1}{3} \cdot 0,714 \right) + \frac{1}{2} \cdot 2 \text{ m} \cdot M_{DC} \cdot \left(\frac{1}{3} \cdot 0,428 + \frac{2}{3} \cdot 0,714 \right) \dots \\ + \frac{1}{2} \cdot 2 \text{ m} \cdot M_{DA} \cdot \left(\frac{2}{3} \cdot 0,714 + \frac{1}{3} \cdot 1 \right) + \frac{1}{2} \cdot 2 \text{ m} \cdot M_{AD} \cdot \left(\frac{1}{3} \cdot 0,714 + \frac{2}{3} \cdot 1 \right) \end{array} \right]$$

$$\varphi_{1p} = 6.45 \times 10^{-3}$$

$$\varphi_{1t} = \Delta_{1\Delta t} \quad \varphi_{1t} = 2.142 \times 10^{-3}$$

$$\varphi_{1\Delta} = -(1 \cdot 0.01 - 0.143 \cdot 0.01) \quad \varphi_{1\Delta} = -8.57 \times 10^{-3}$$

$$\varphi_I = \varphi_{1p} + \varphi_{1t} + \varphi_{1\Delta} \quad \varphi_I = 2.186 \times 10^{-5}$$

$$\Delta = \frac{\varphi_I}{\Delta_{1p}} \quad \Delta = 0.071 \%$$